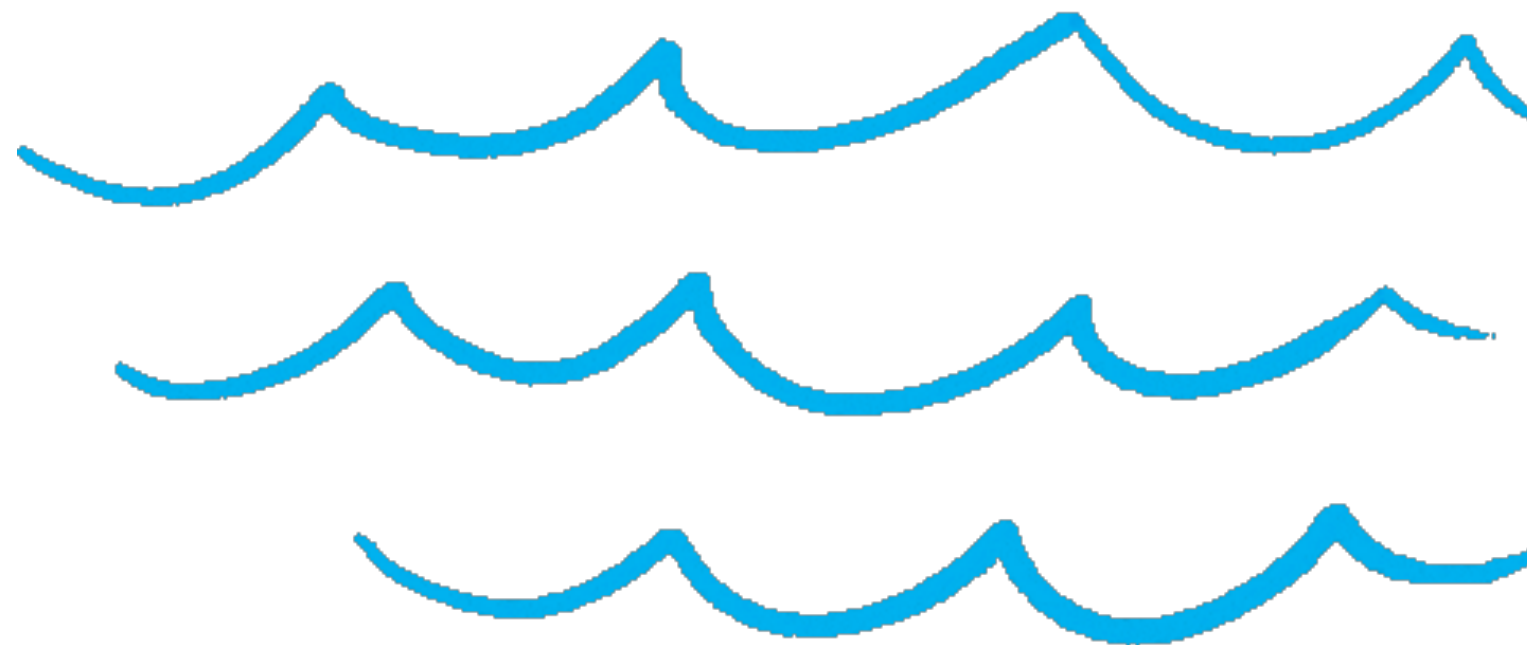
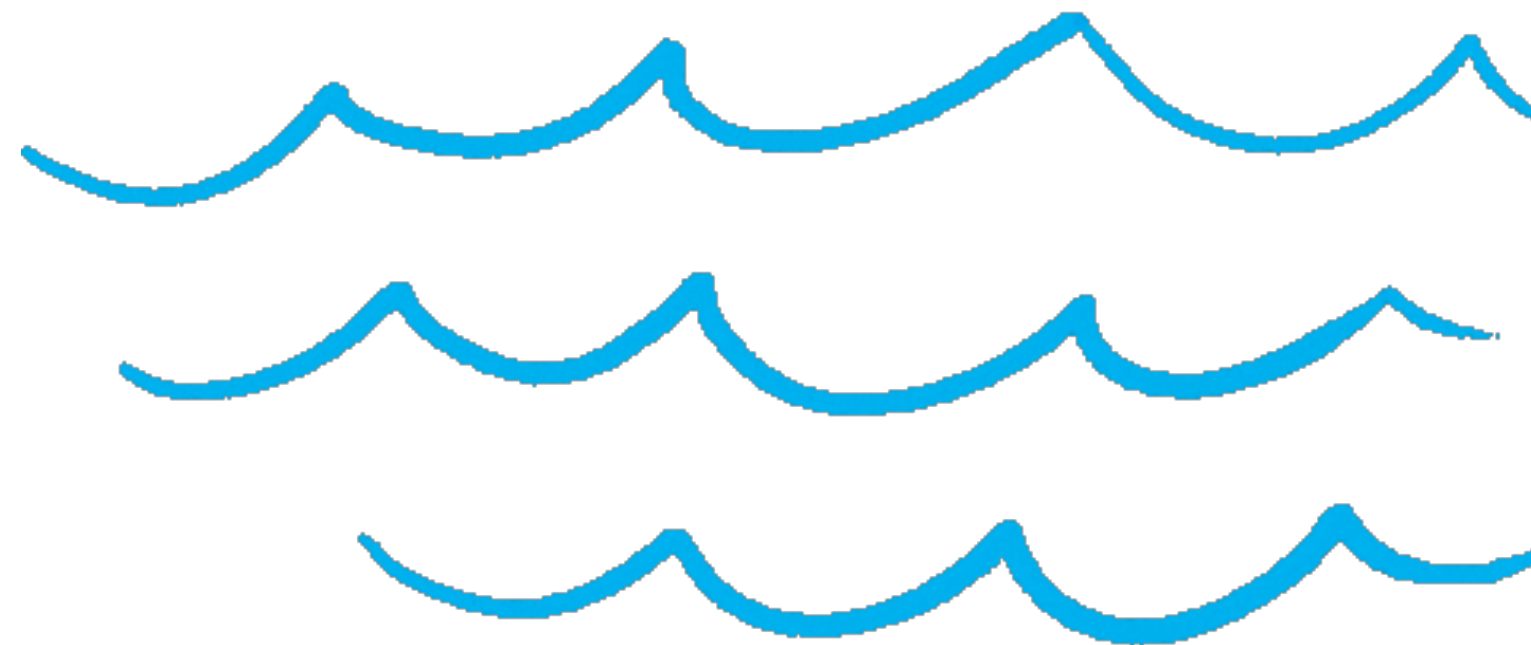


Asteroseismology Crash Course

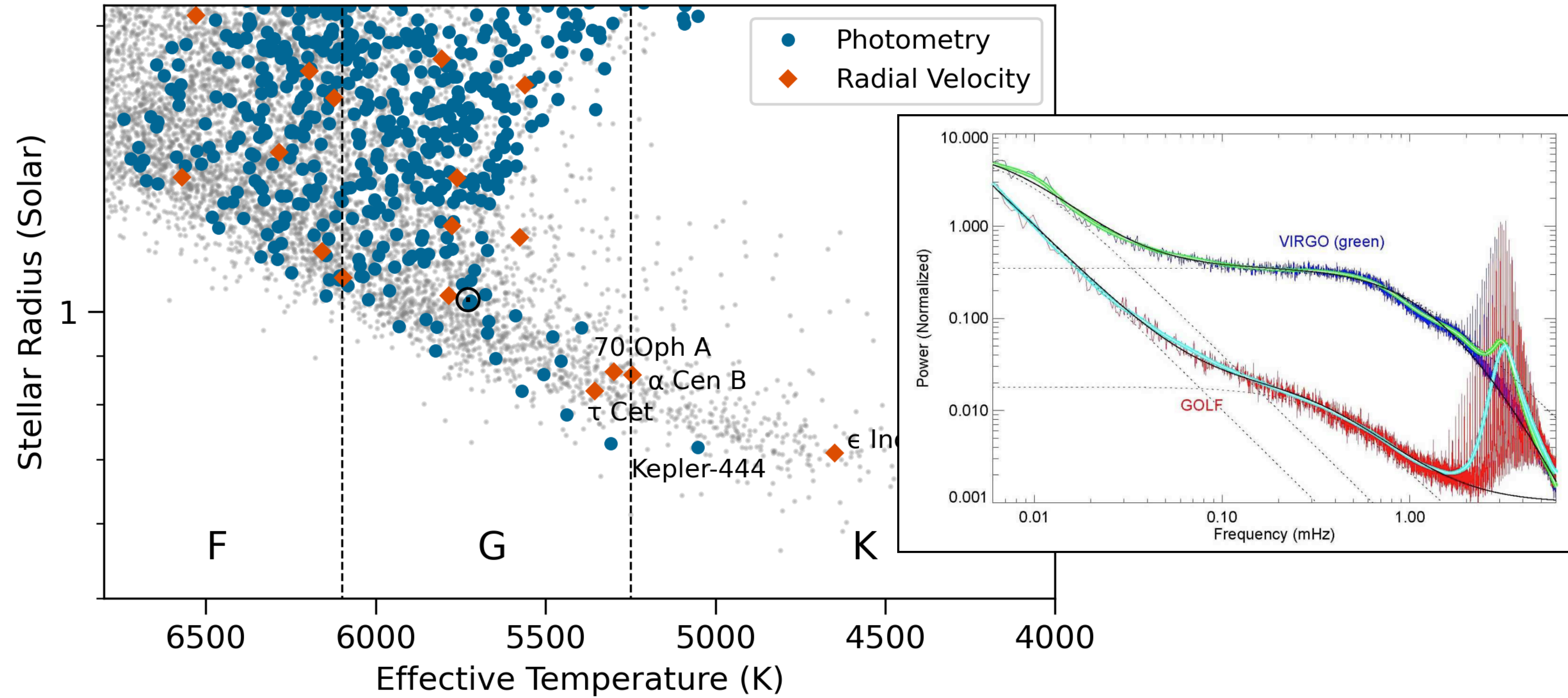
Observational Analysis



A Practical Guide to Stellar Age

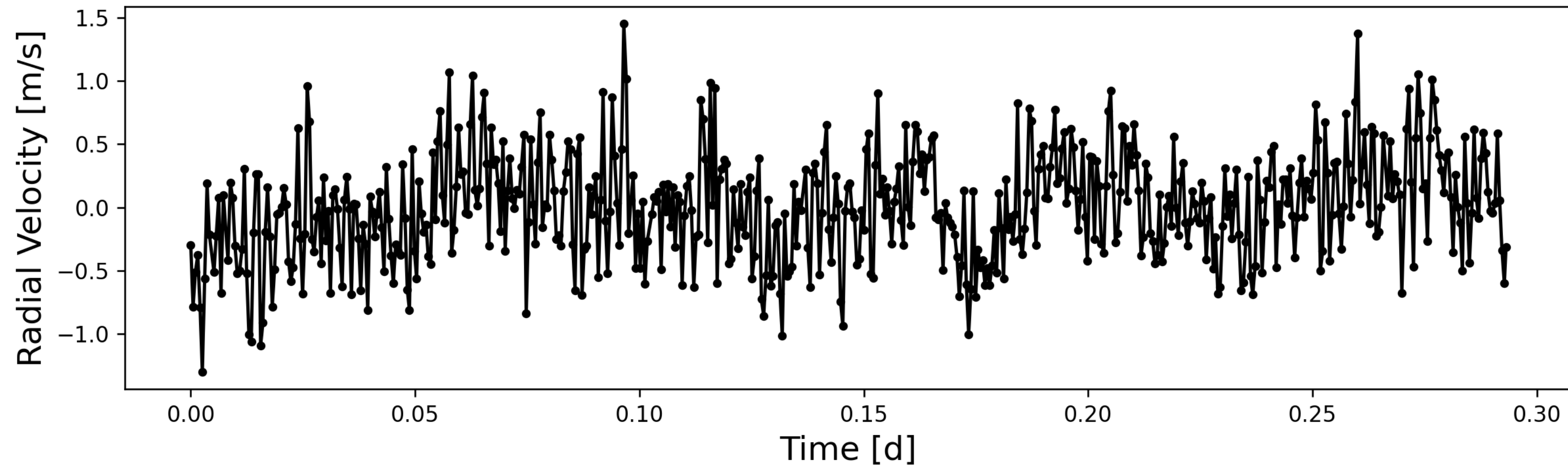


Observations

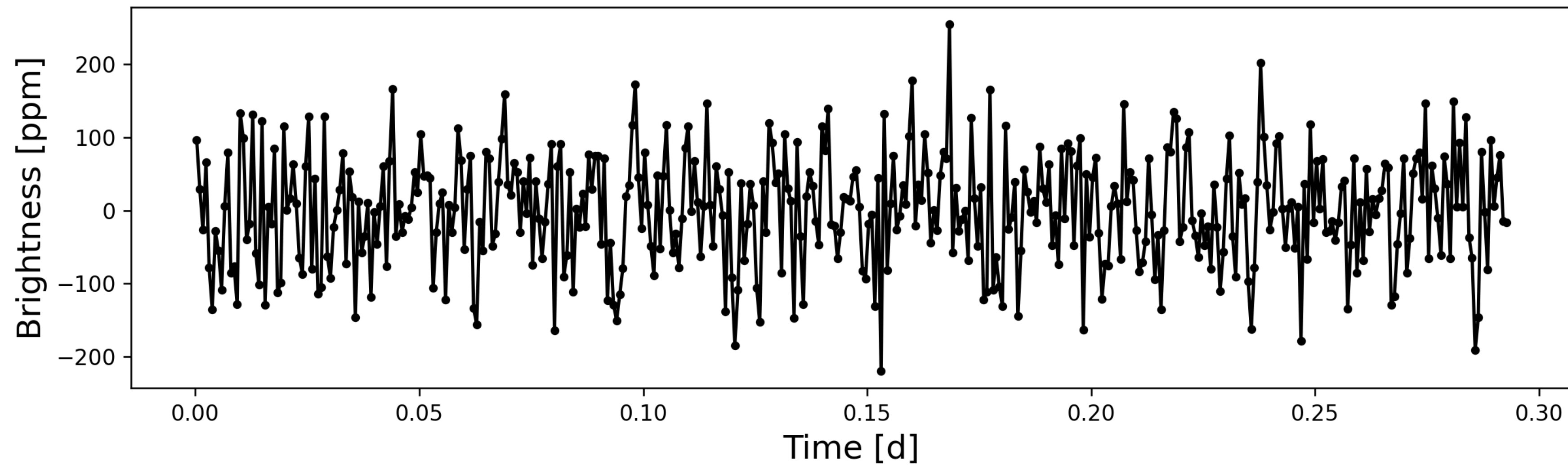


Time Series

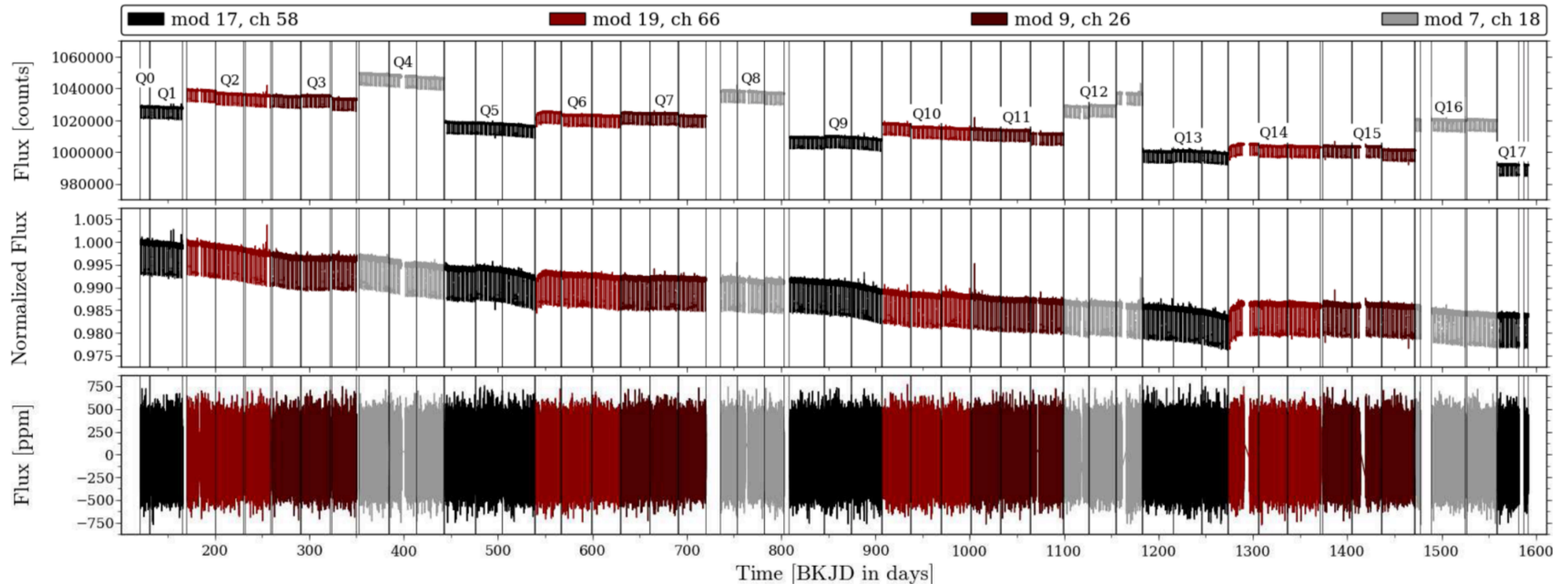
Keck/KPF



TESS

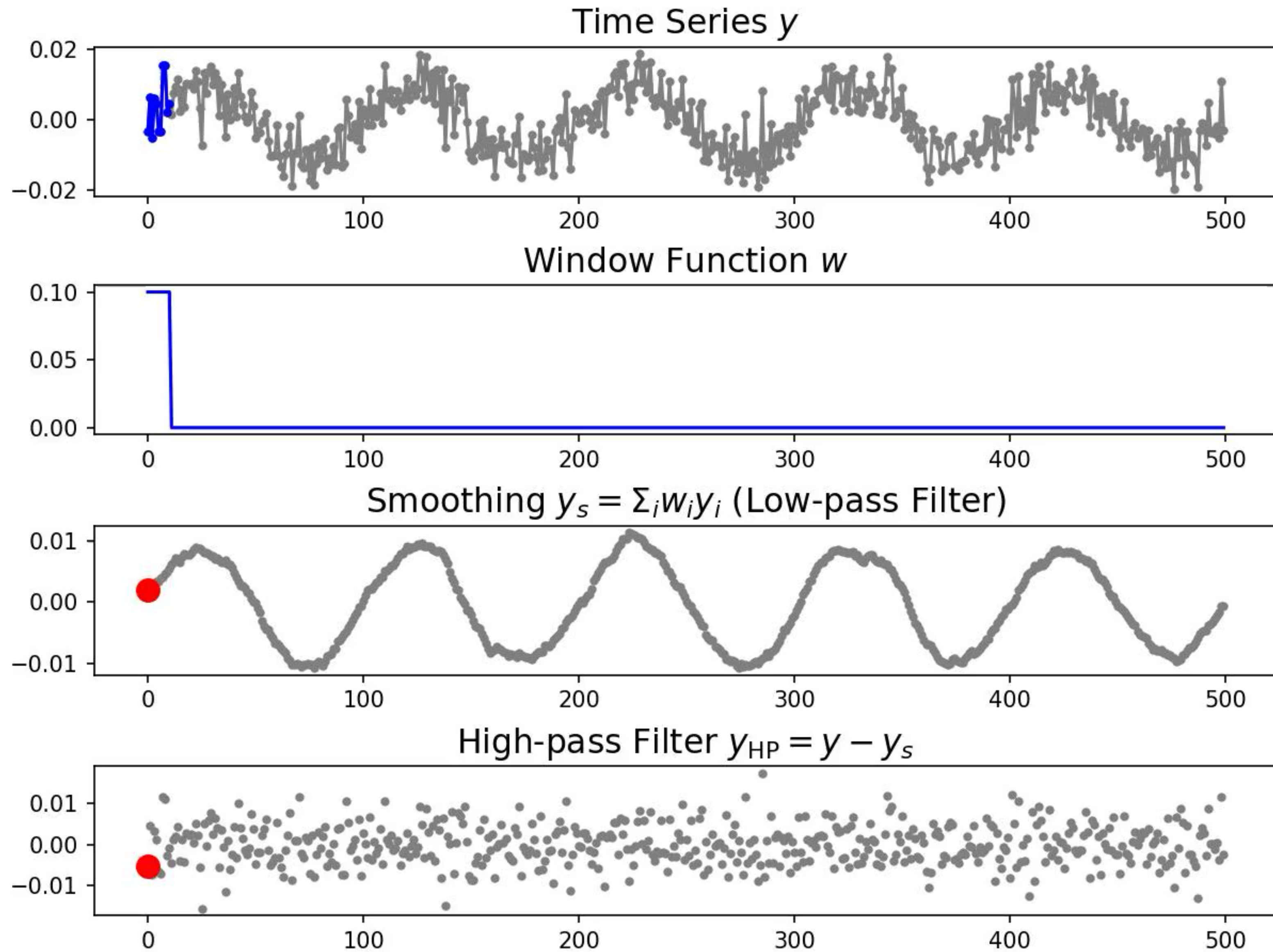


Time Series Preparation

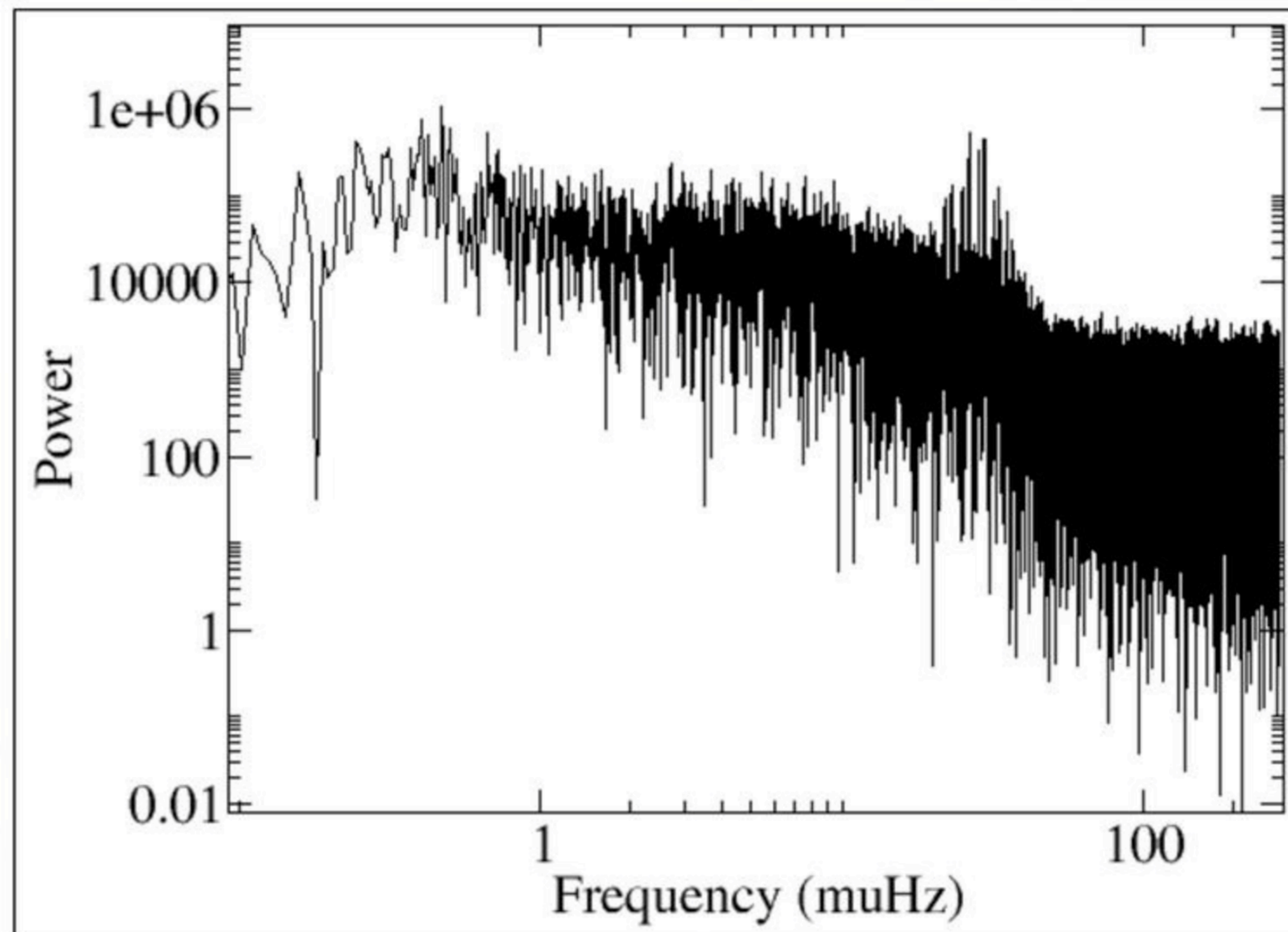


Apply high-pass filter to remove sharp time-domain features that could cause power leakage to high frequencies (where oscillations live).

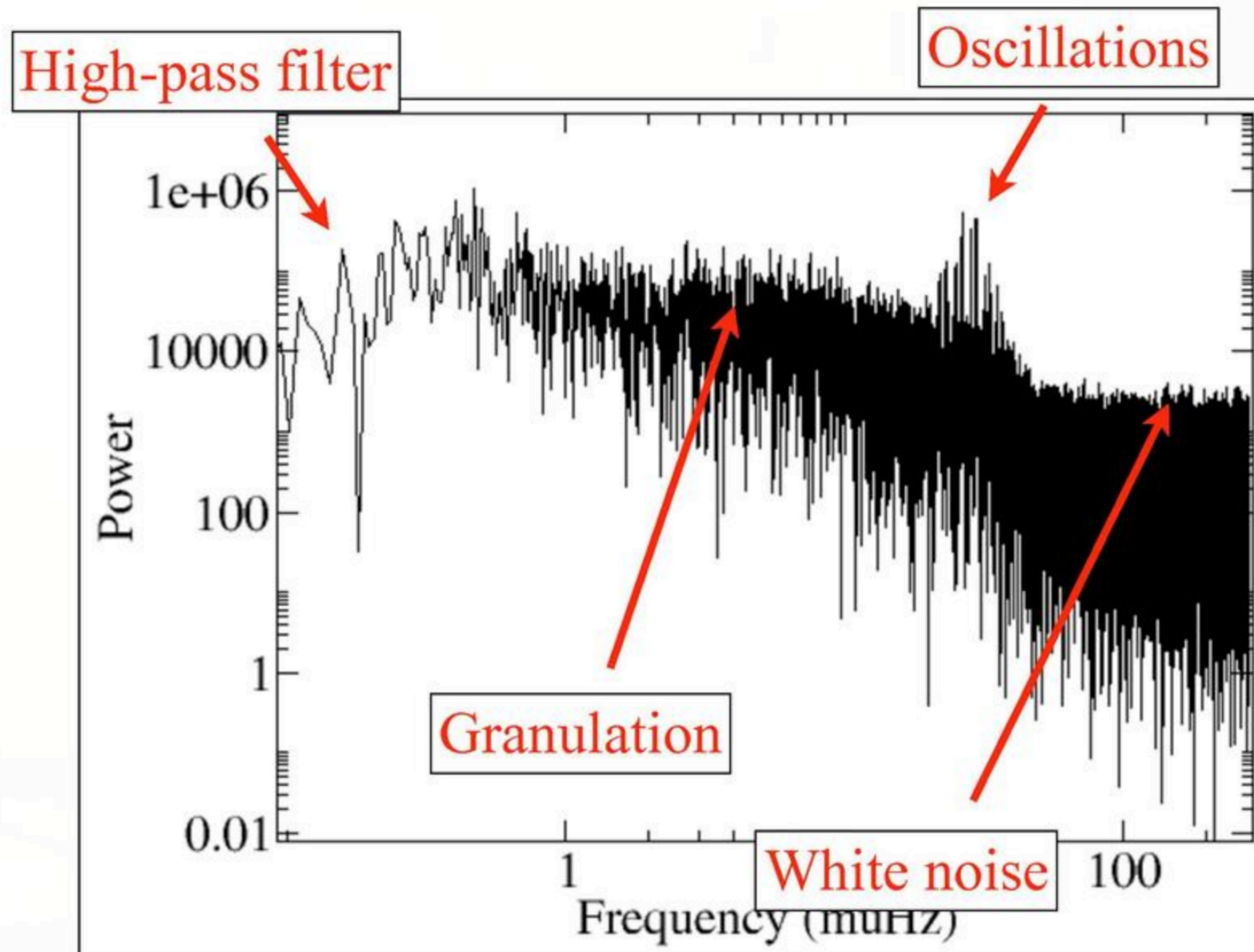
High-pass filter



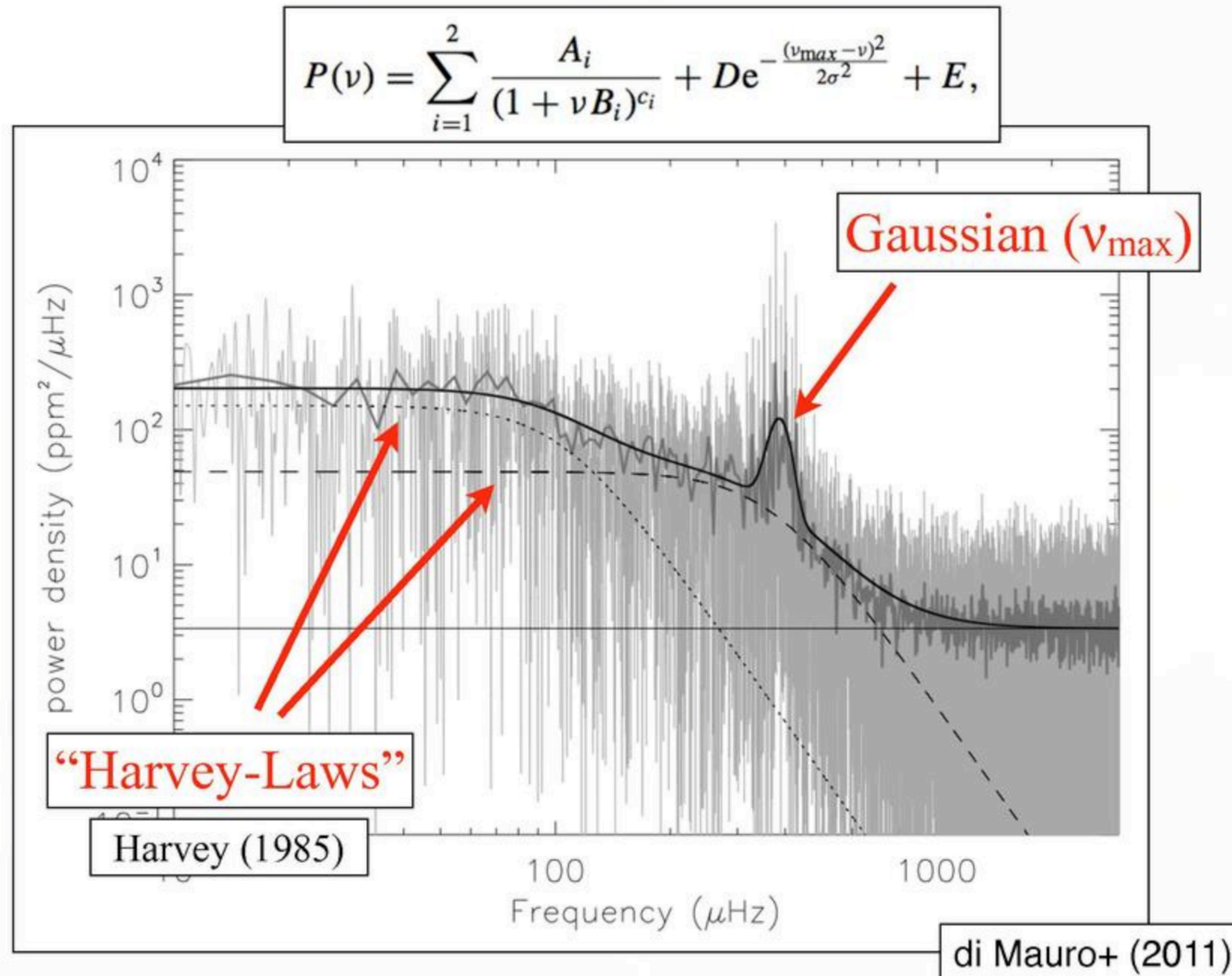
Power Spectrum



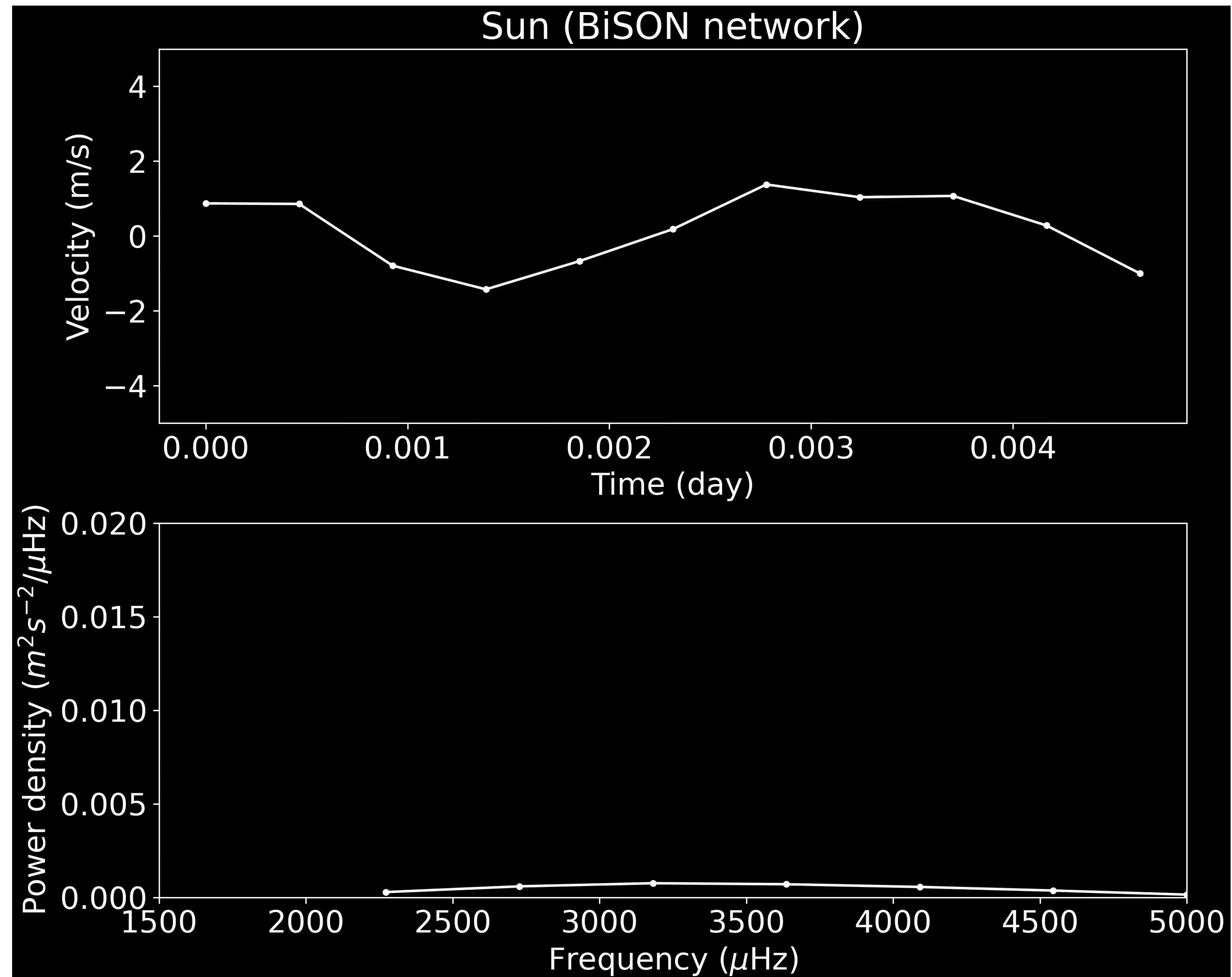
Power Spectrum



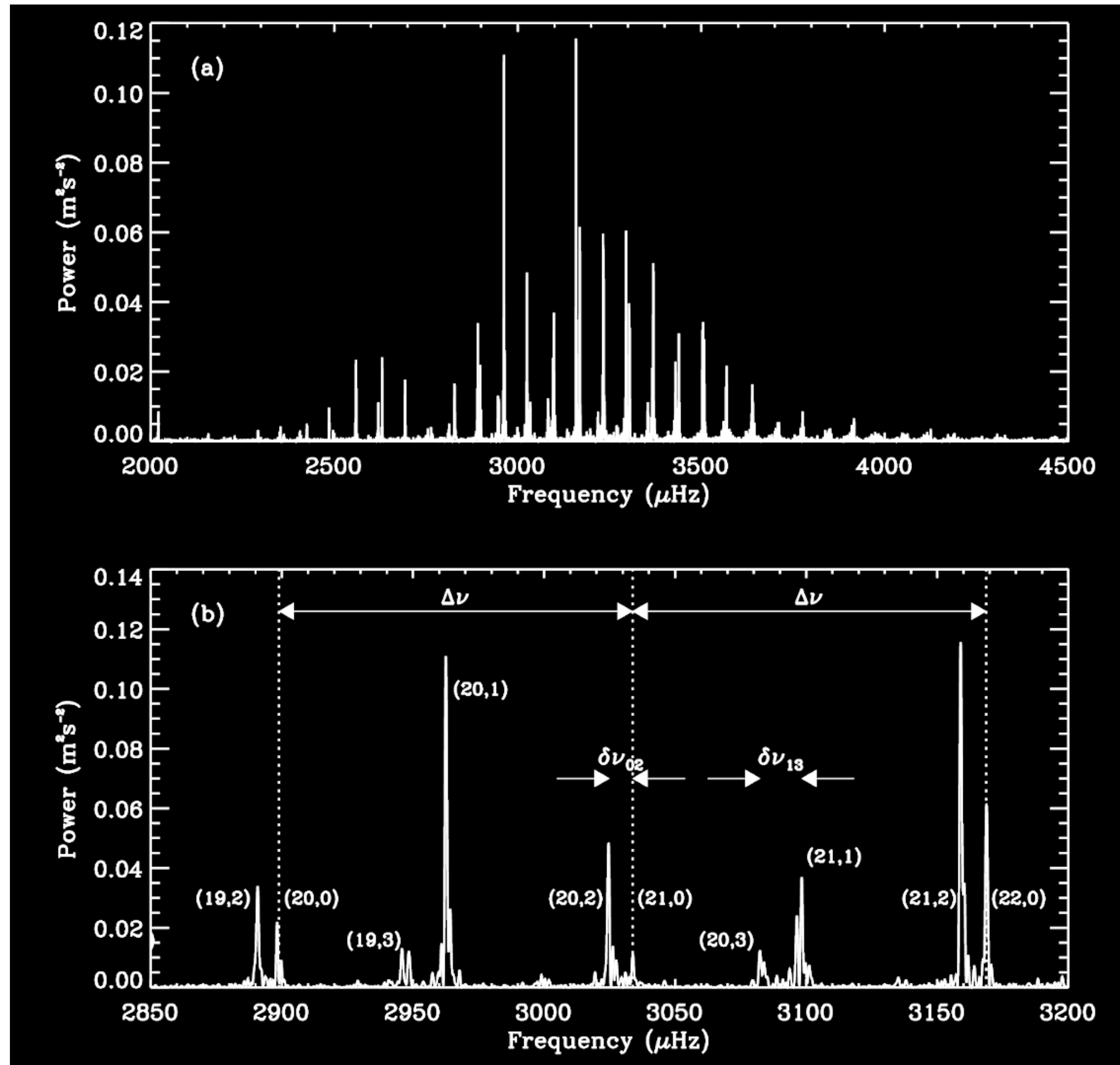
Power Spectrum



Oscillations in the Sun



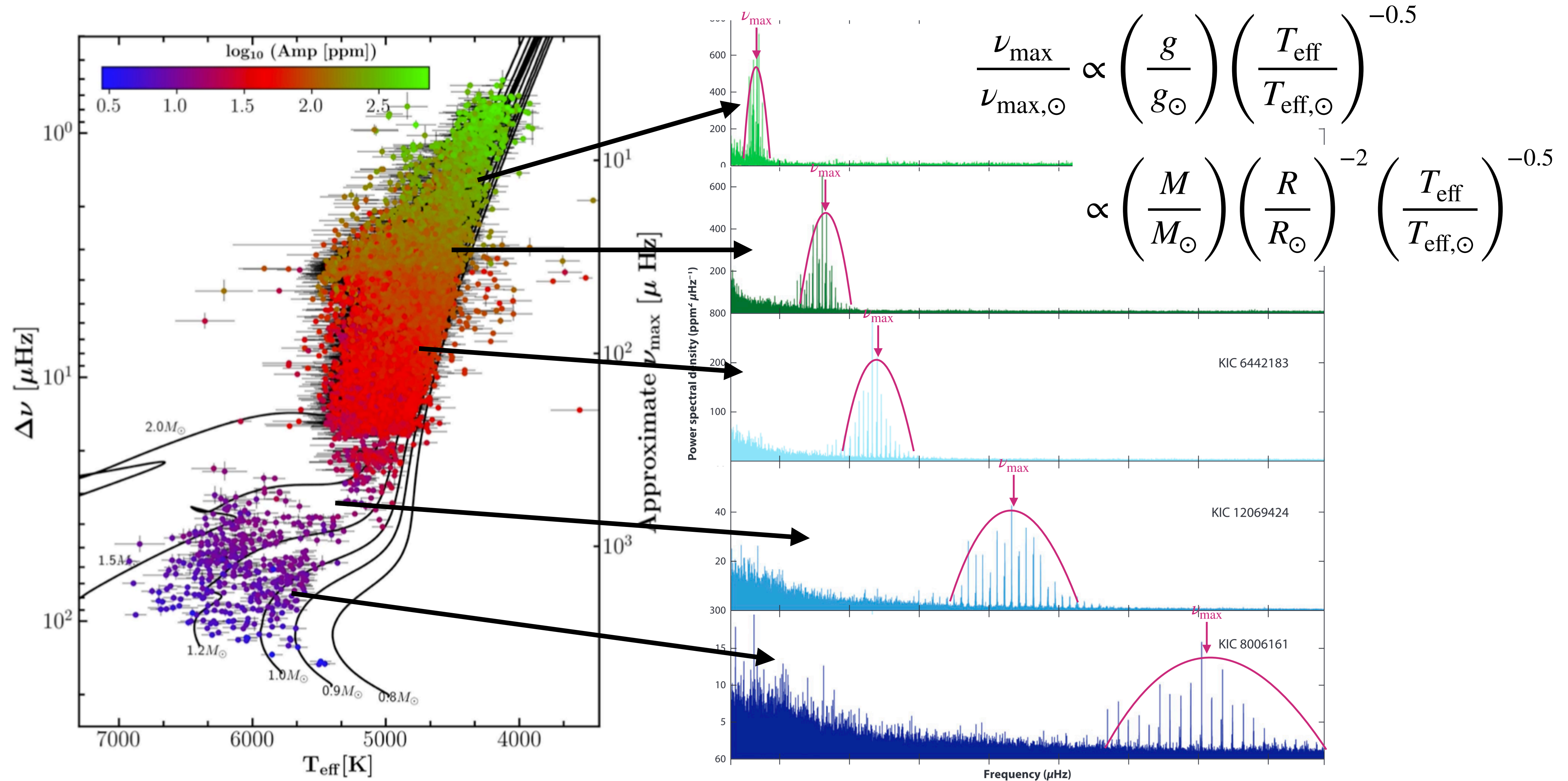
Oscillations in the Sun



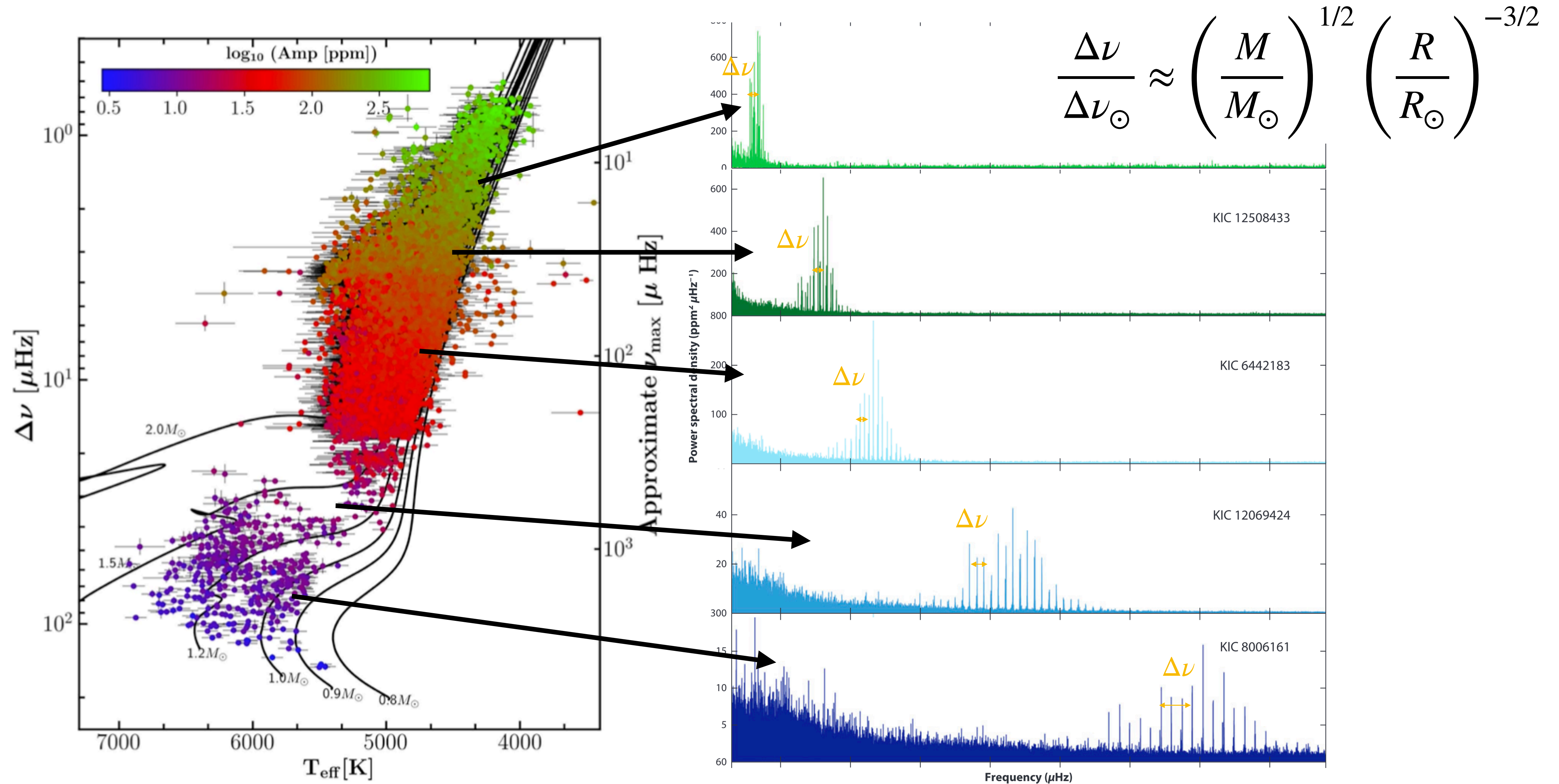
Mode Identification ($n, l, *m$)

- Cool stars (Solar-like oscillators, main-sequence dwarfs and red giants):
 - Frequency/period spacing patterns
- Hot stars (classical pulsators)
 - Frequency/period spacing patterns
 - Multi-colour photometry

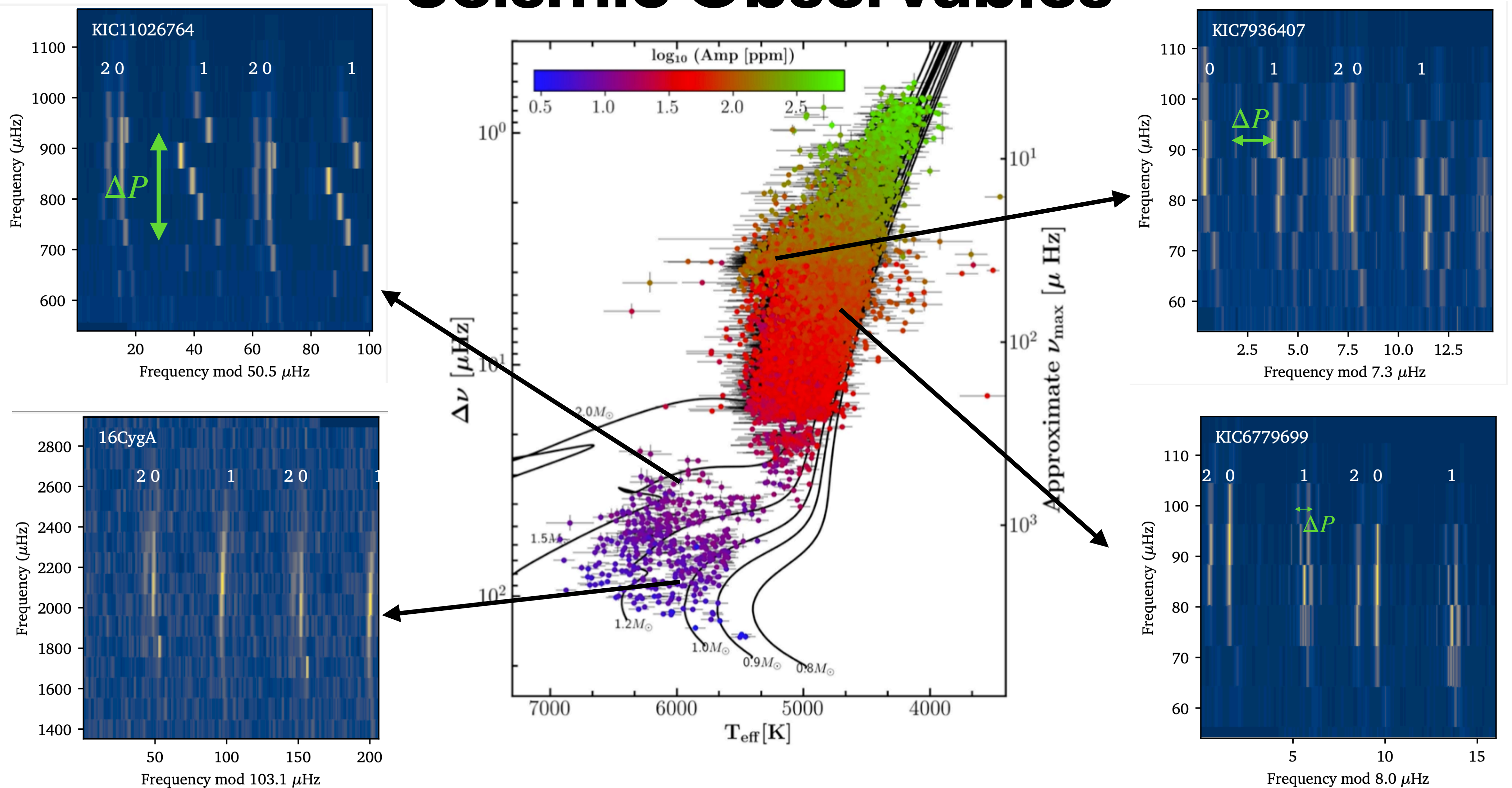
Seismic Observables



Seismic Observables



Seismic Observables



Extracting mode parameters

- Prewhitening (iterative sine-wave fitting)
- Gaussian Process (time domain machine learning stuff)
- Peakbagging (fourier domain mode fitting)

Peakbagging

Peak bagging or hill bagging is an activity in which hikers, climbers, and mountaineers attempt to reach a collection of summits, published in the form of a list.

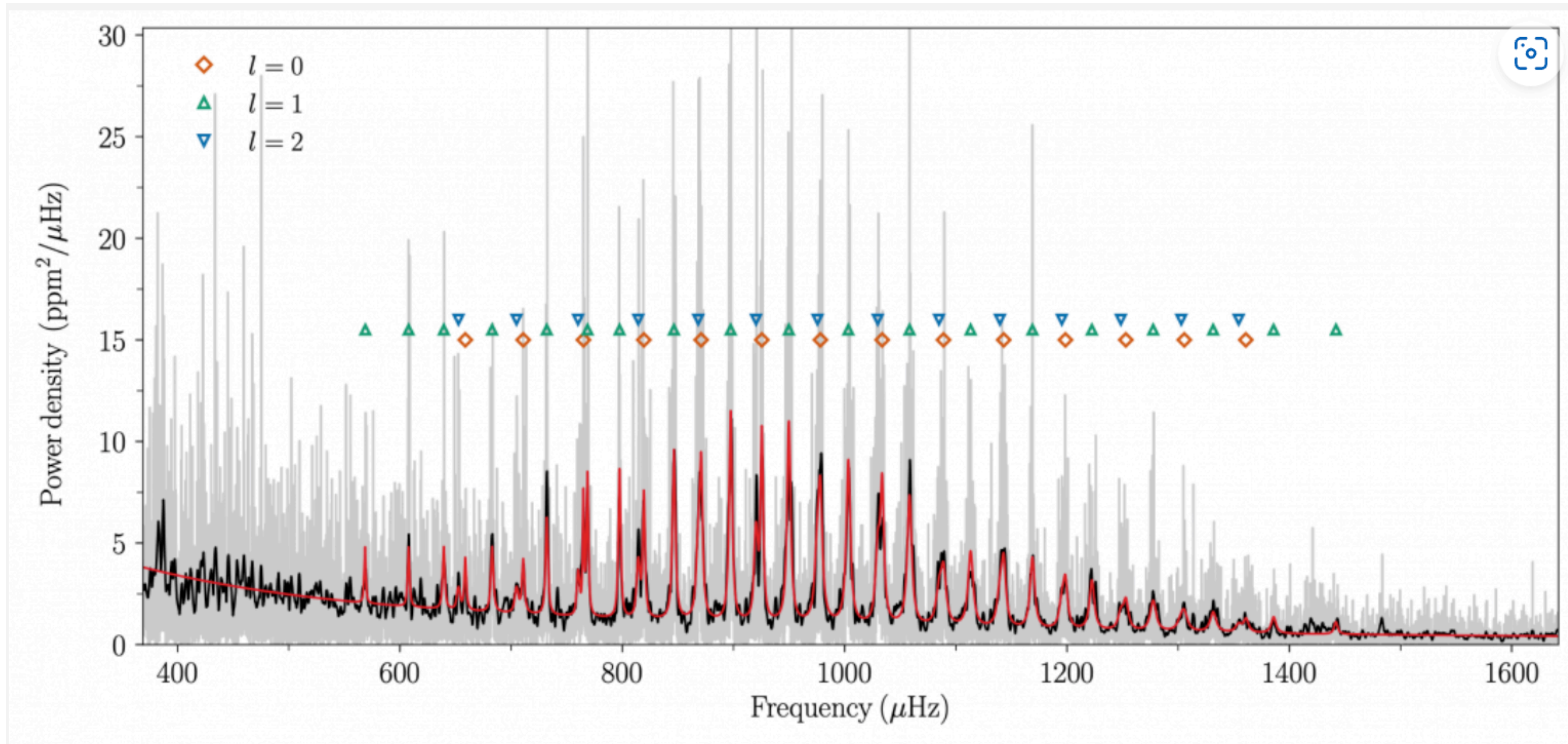


Wikipedia

https://en.wikipedia.org/wiki/Peak_bagging ⋮

[Peak bagging - Wikipedia](https://en.wikipedia.org/wiki/Peak_bagging)

Peakbagging



Fitting Fundamentals

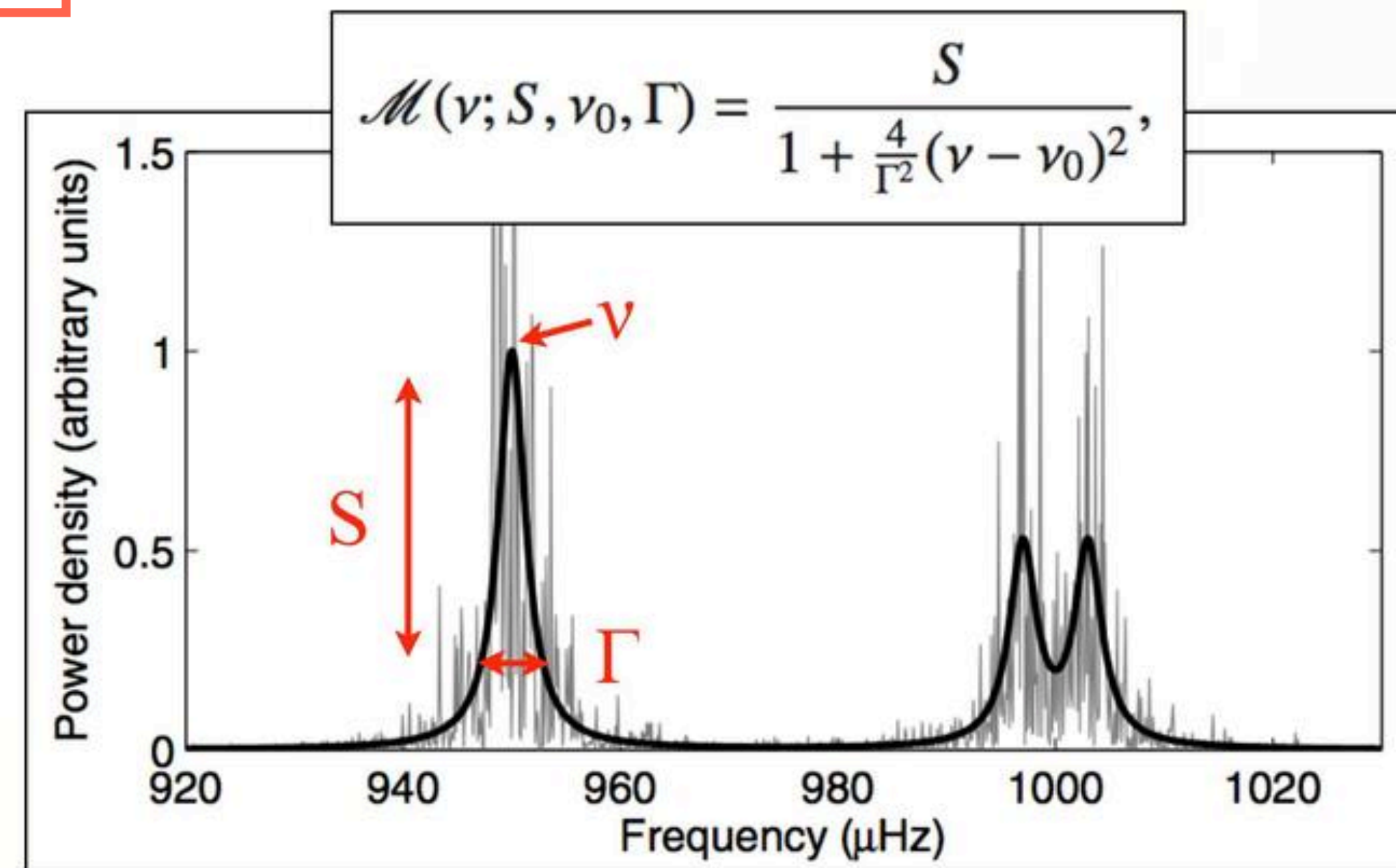
Bayesian Framework

- Data (power spectrum)
- Model (the approximate and parameterized model)
- Bayesian theorem

$$\bullet \quad p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)} \quad \text{Posterior} = \frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}$$

Power Spectrum Model

$$p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)}$$



For each radial mode:

frequency, height, width

For each non-radial
mode:

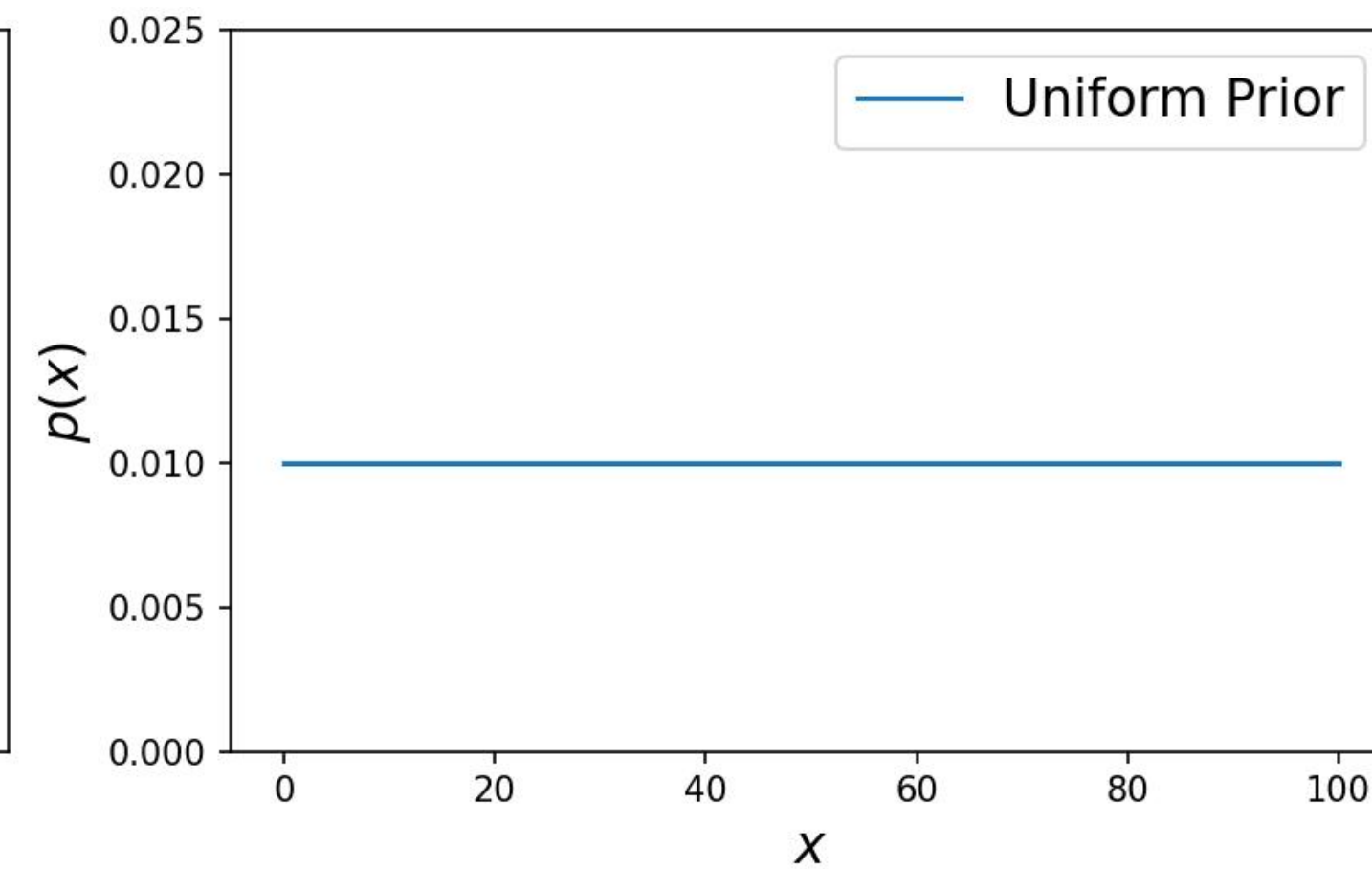
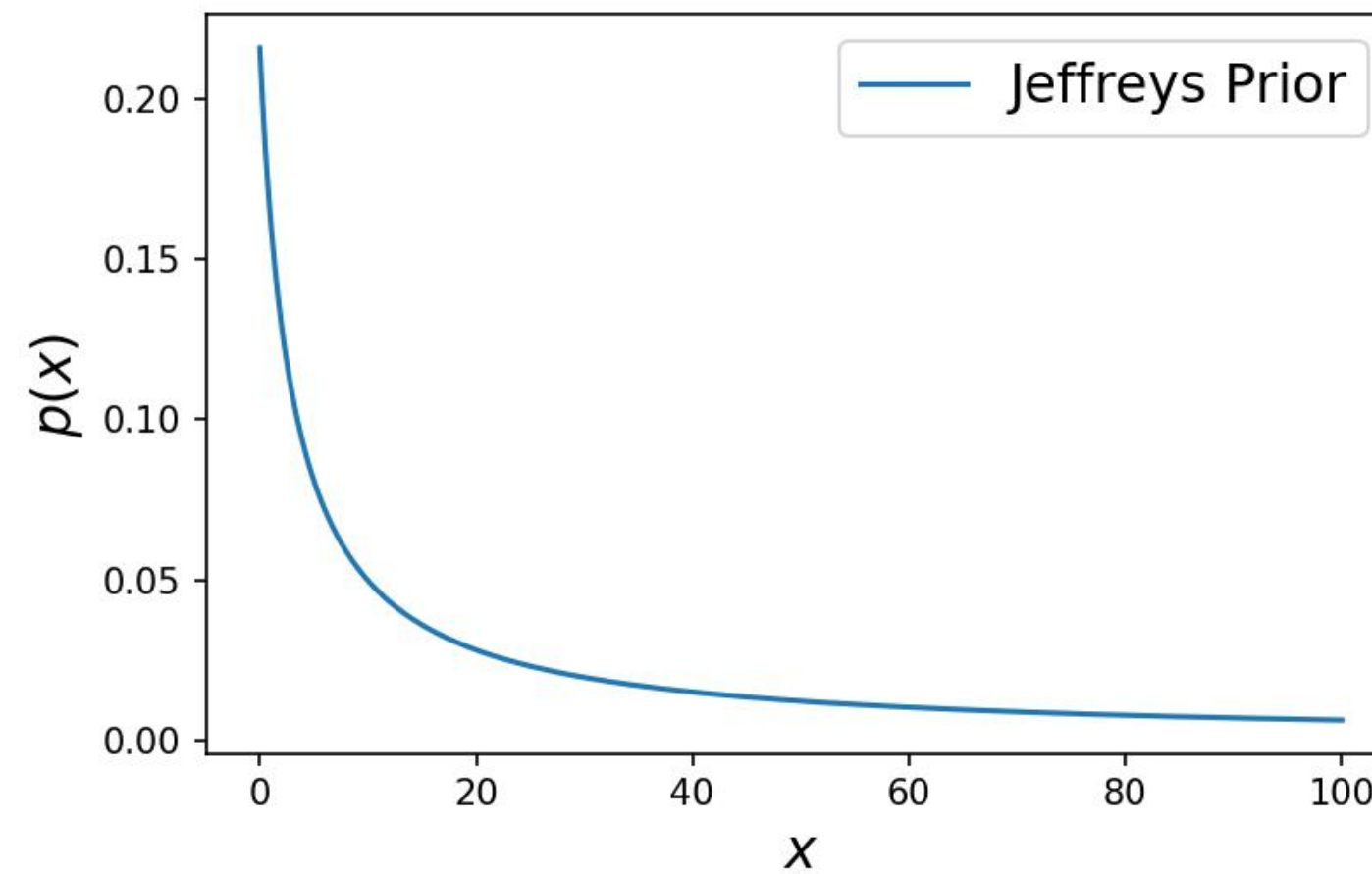
frequency, height, width

[splitting, inclination] [magnetic field strength!]

Prior

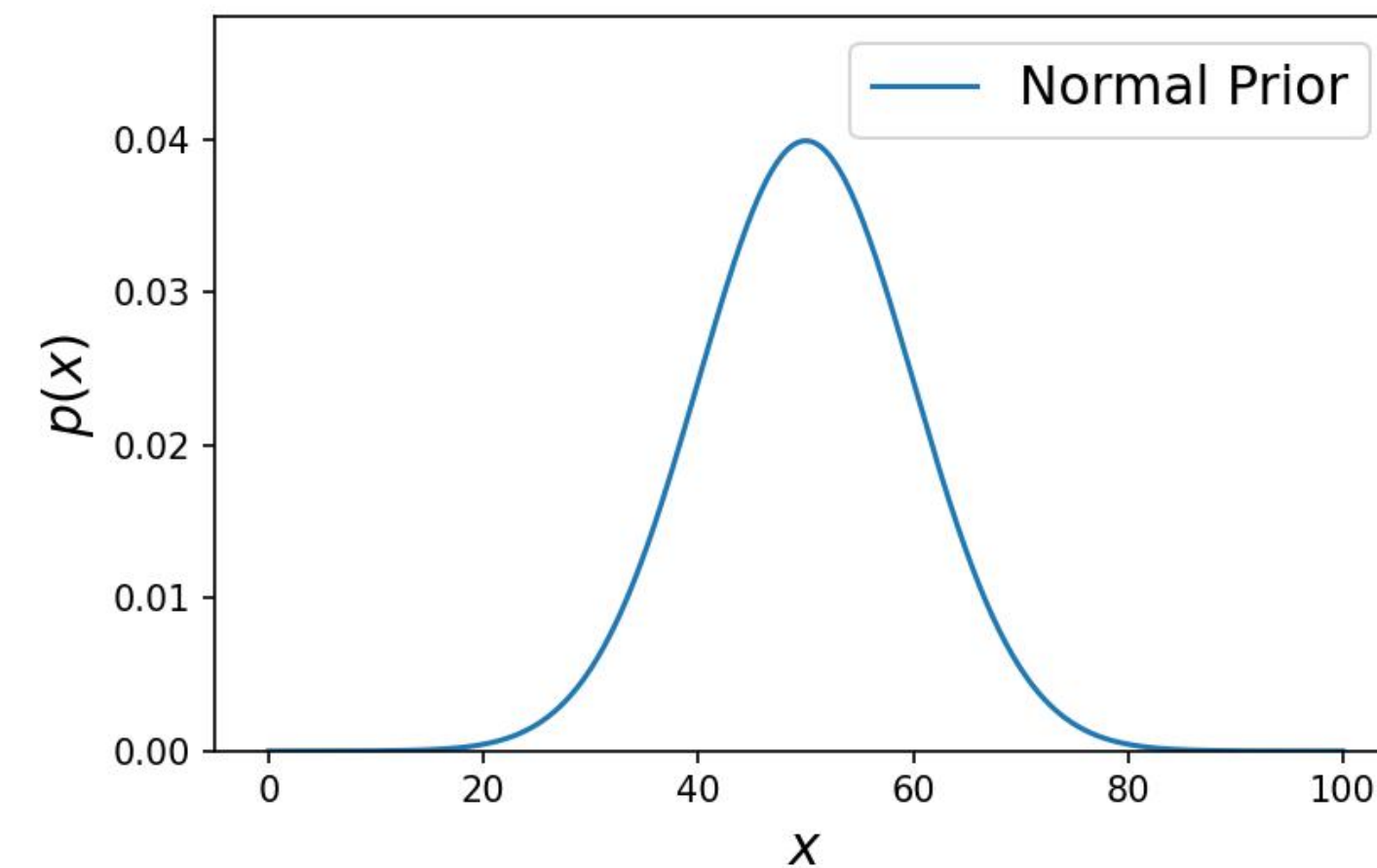
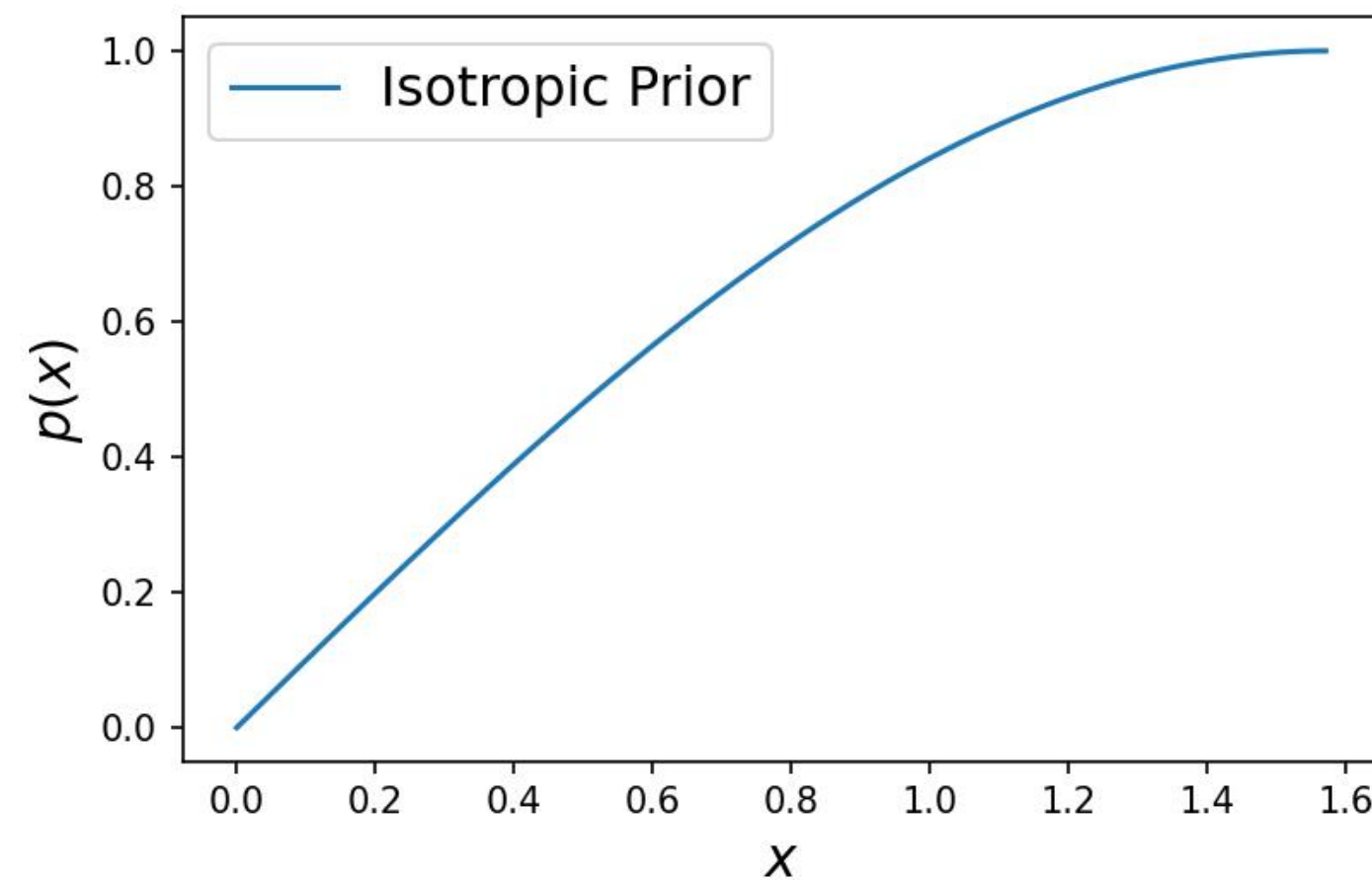
$$p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)}$$

Linewidths, amplitudes



Frequencies

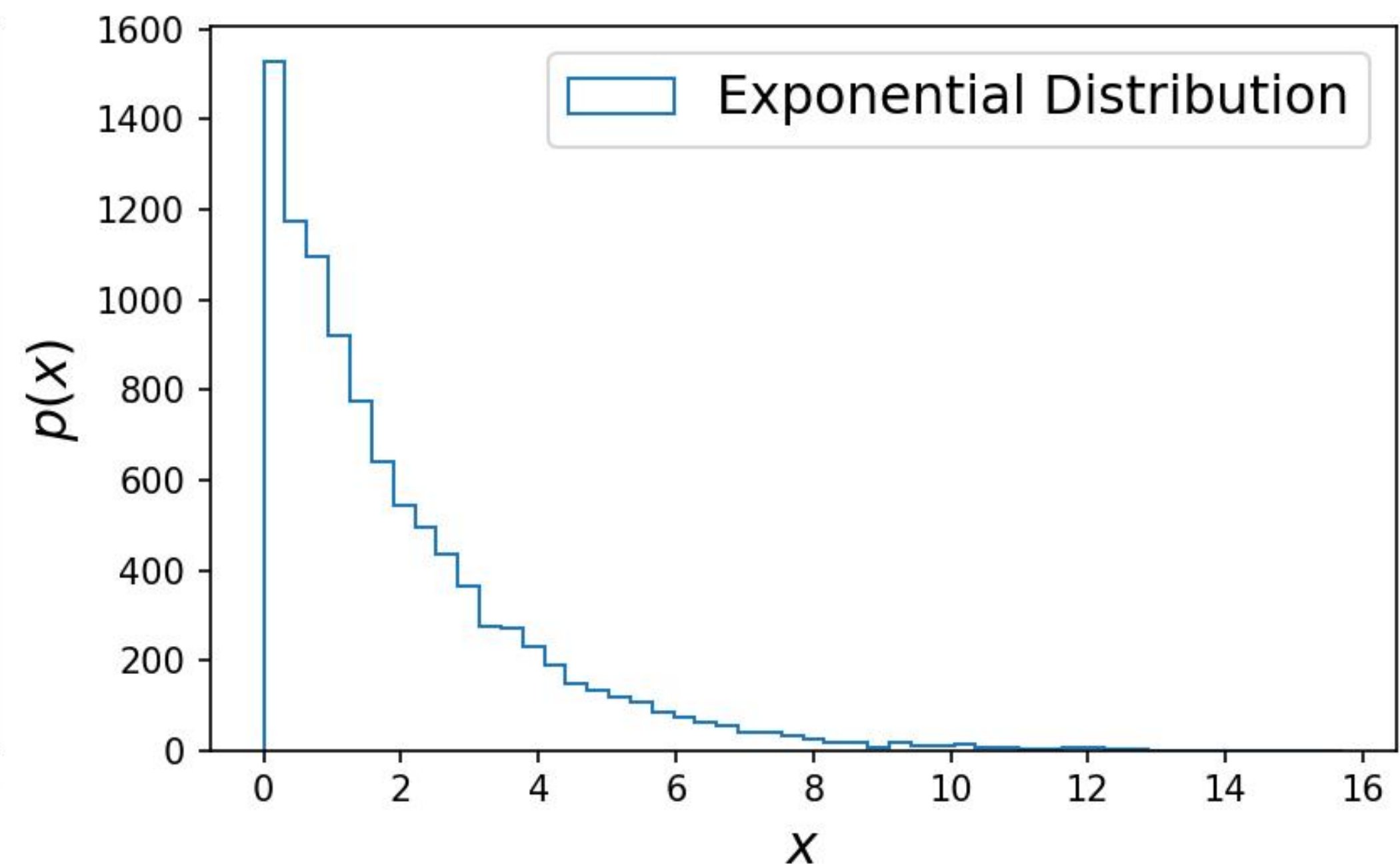
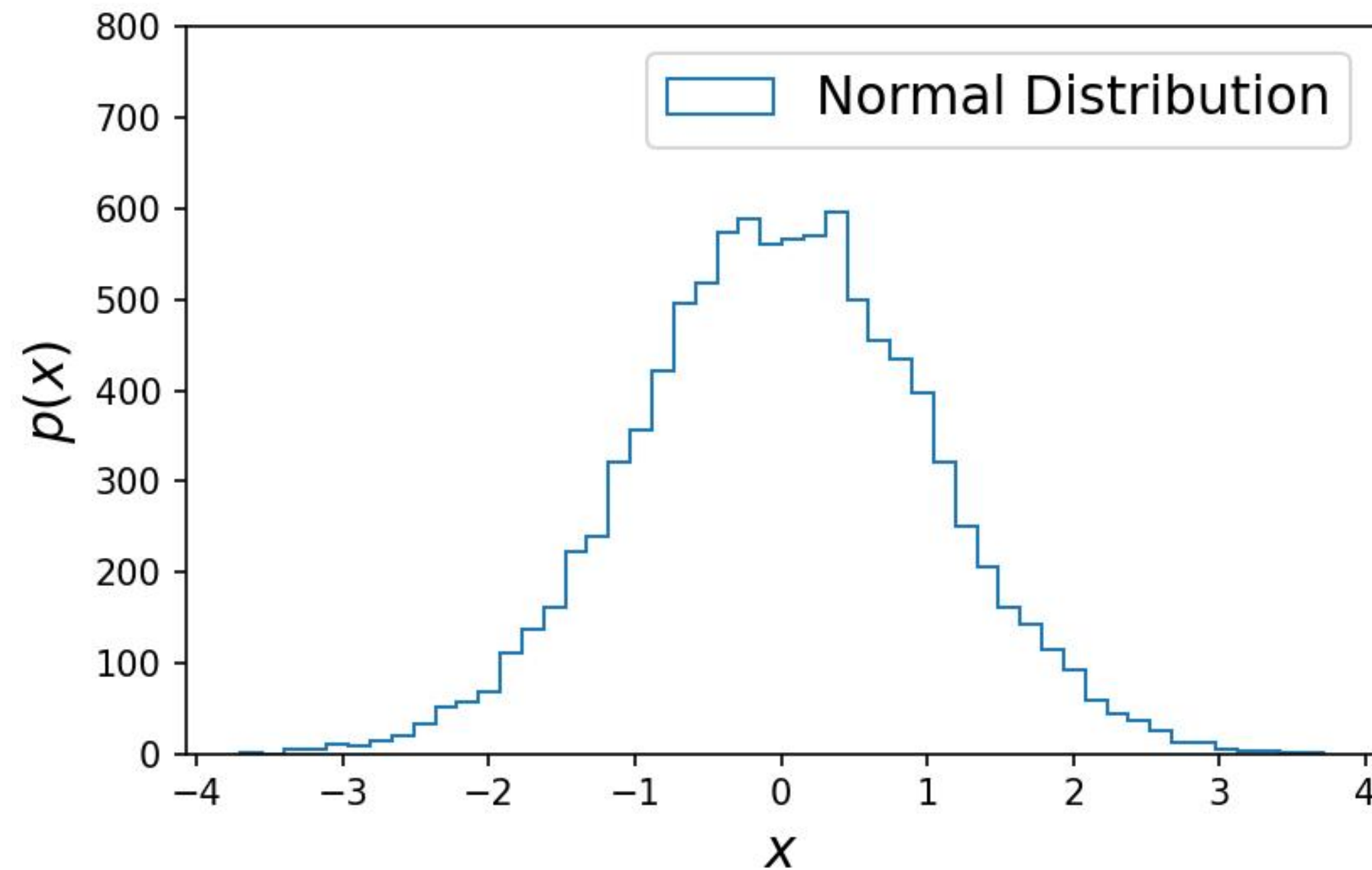
Inclinations



Use with caution!

Likelihood

$$p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)}$$



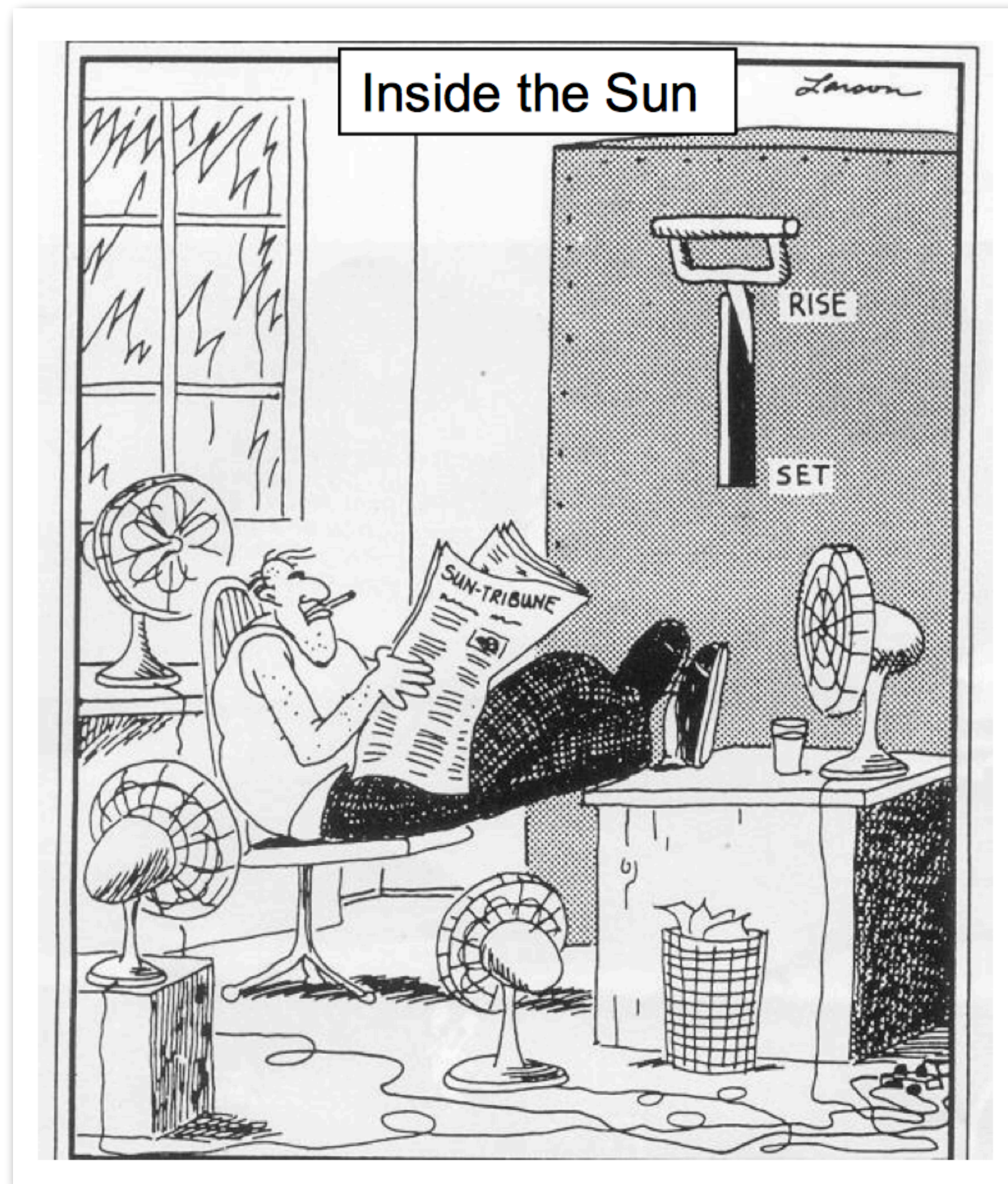
$$\frac{(D_i - M_i)}{\sigma_i} \sim \mathcal{N}(0,1) \Rightarrow \ln L(\theta) = -\chi^2/2 = -\sum_i \frac{[D_i - M_i(\theta)]^2}{2\sigma_i^2}$$

$$\frac{D_i}{M_i} \sim \chi_{k=2}^2 \Rightarrow \ln L(\theta) = -\chi^2/2 = -\sum_i \left[\ln M_i(\theta) + \frac{D_i}{M_i(\theta)} \right]$$

Posterior and Sampling

- Posteriors $p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)} \propto p(\theta | M) p(D | \theta, M)$
- MCMC sampling, e.g.
 - emcee (ensemble sampling): <https://emcee.readthedocs.io/en/stable/>
 - dynesty (nested sampling): <https://dynesty.readthedocs.io/en/stable/>
 - PyMC (HMC sampling): <https://www.pymc.io/welcome.html>

Goals of Asteroseismology



- Understand internal physics and structure
- Measure stellar properties (mass, radius, age)

-> **asteroseismic model fitting**

Model Construction

MESA & GYRE

 / Modules for Experiments in Stellar Astrophysics

 [Edit on GitHub](#)

Modules for Experiments in Stellar Astrophysics

This site provides documentation for Modules for Experiments in Stellar Astrophysics (MESA), an open-source 1D stellar evolution code.

 / The GYRE Stellar Oscillation Code

 [Edit on GitHub](#)

The GYRE Stellar Oscillation Code

GYRE is a *stellar oscillation code*. Given an input stellar model, GYRE calculates the eigenfrequencies and eigenfunctions for the normal oscillation modes of the model. These data can be put to a variety of uses; the most common is to compare them against observed oscillation frequencies of a star, allowing constraints on the star's fundamental parameters (mass, radius, etc.) to be established — the discipline of *asteroseismology*.

- Open-source, modular, large community
- MESA summer school (with tutorials): <https://mesahub.github.io/summer-school-2023/agenda/>

Model Construction

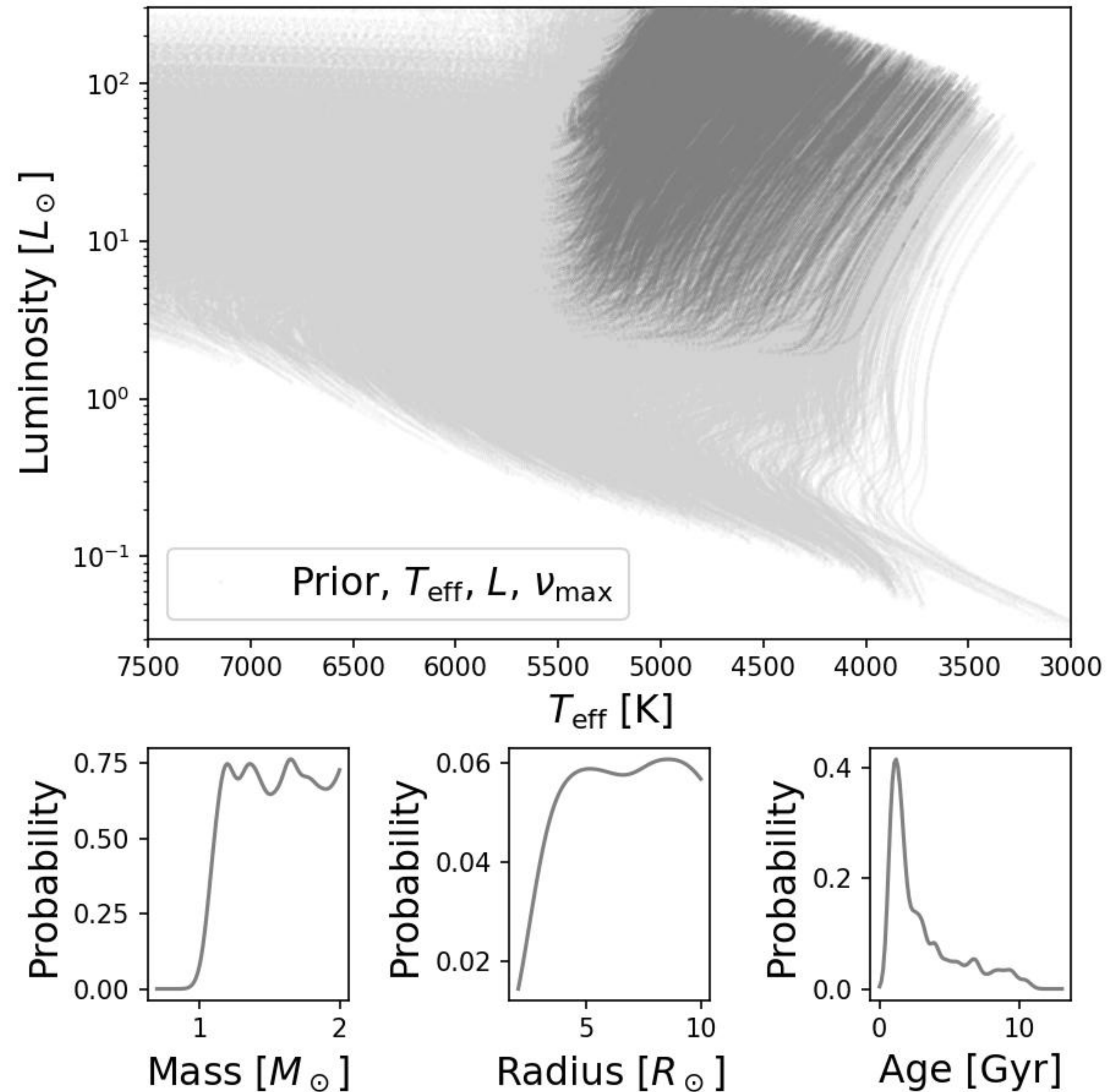
MESA & GYRE



- Microphysics
 - Equation of state
 - Opacities
 - Nuclear reaction networks
- Macrophysics
 - Mixing length theory of convection
 - Convective overshoot mixing
 - Atmospheric boundary condition
 - Diffusion and gravitational settling
 - Mass loss and accretion
- Initial conditions
 - Initial mass
 - Initial composition (Y, Z)

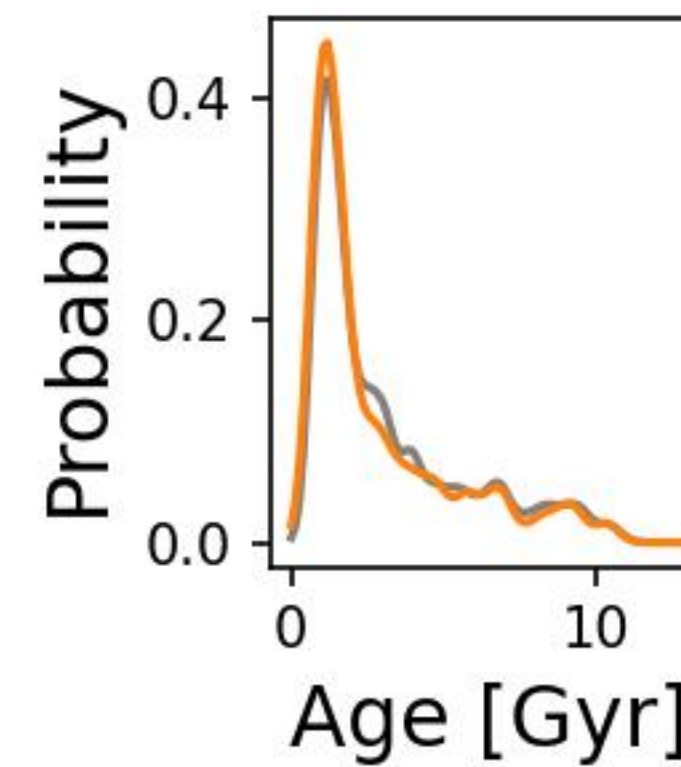
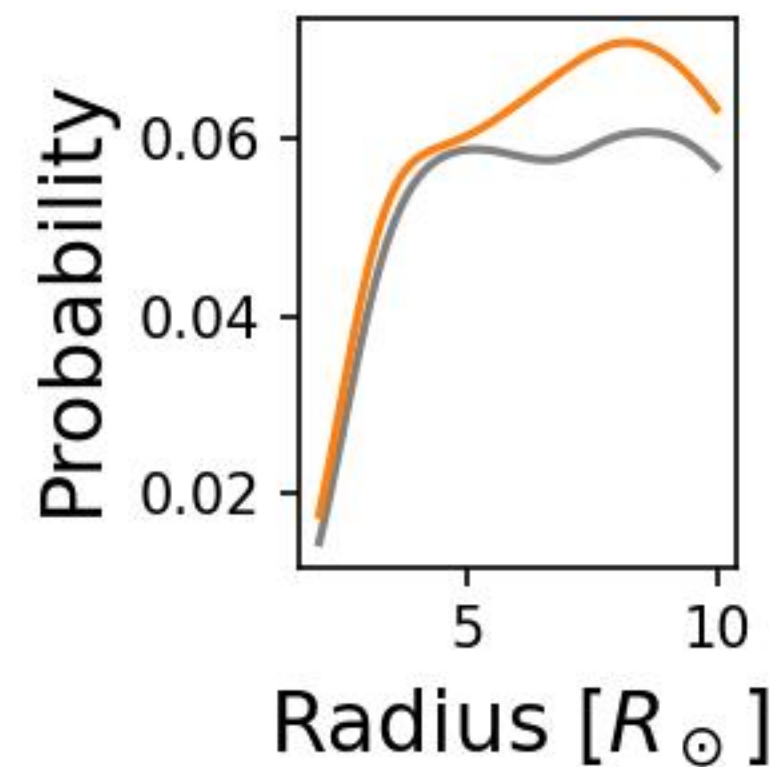
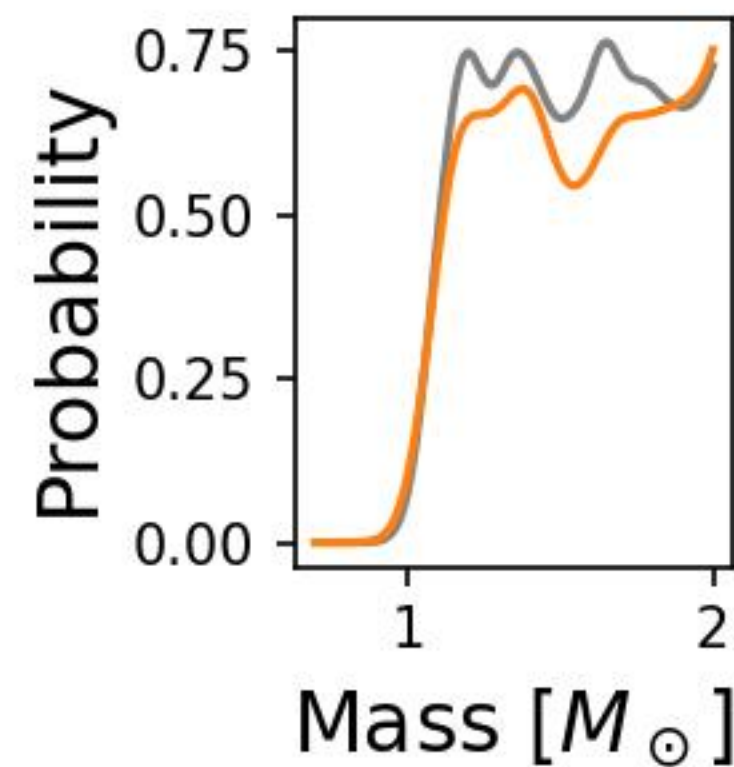
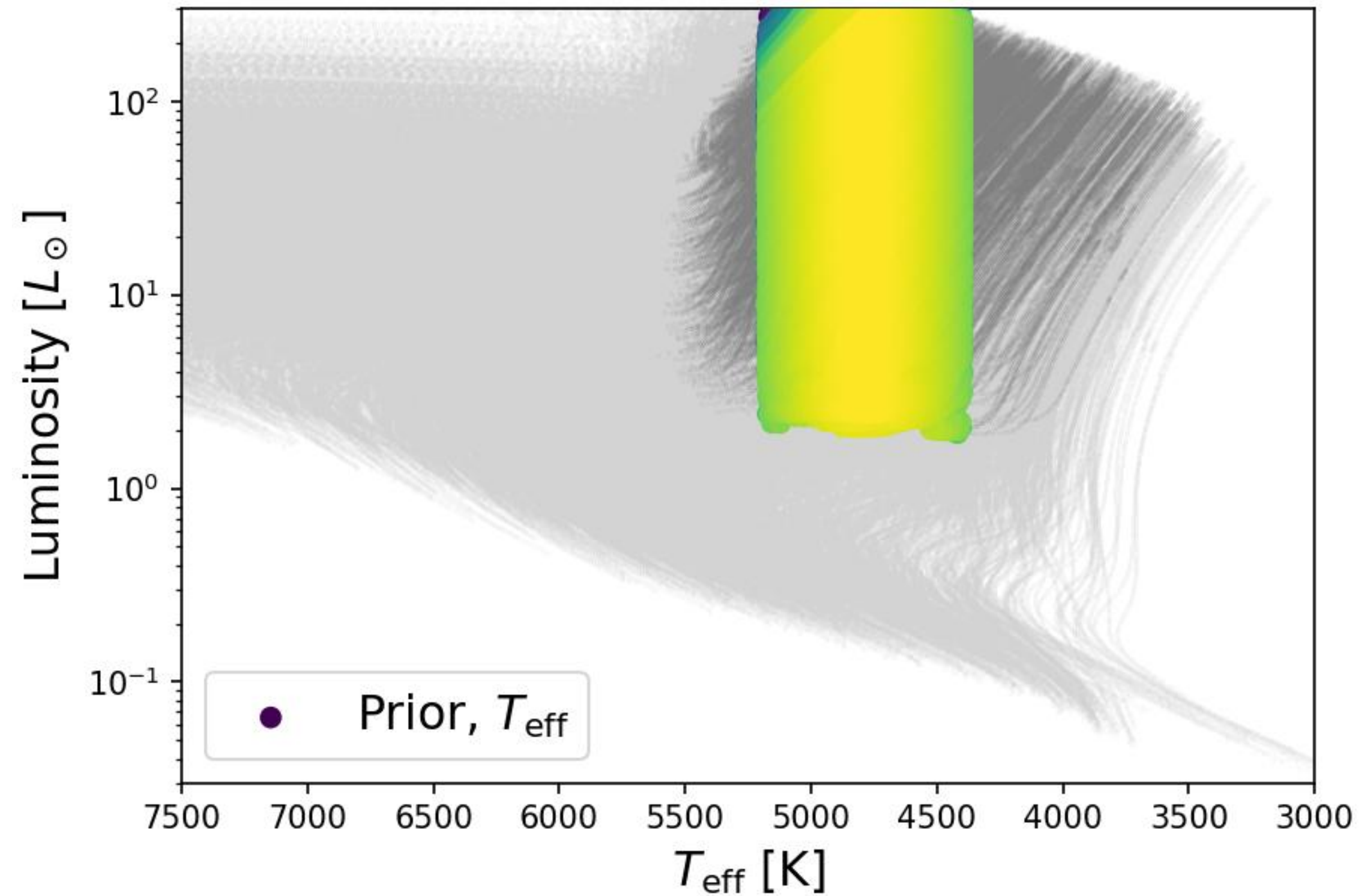
Constraints with asteroseismology

$M, Z, Y, \alpha_{\text{MLT}}, \tau$



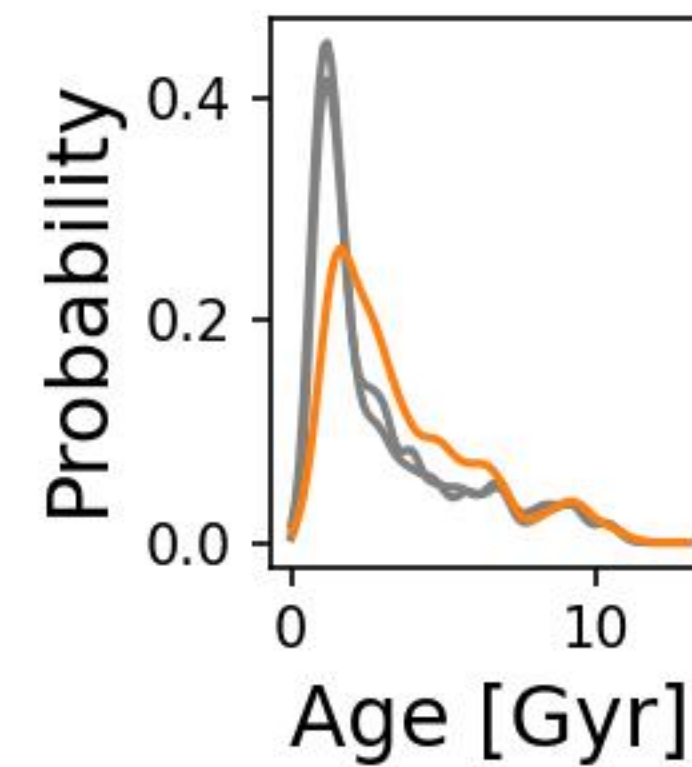
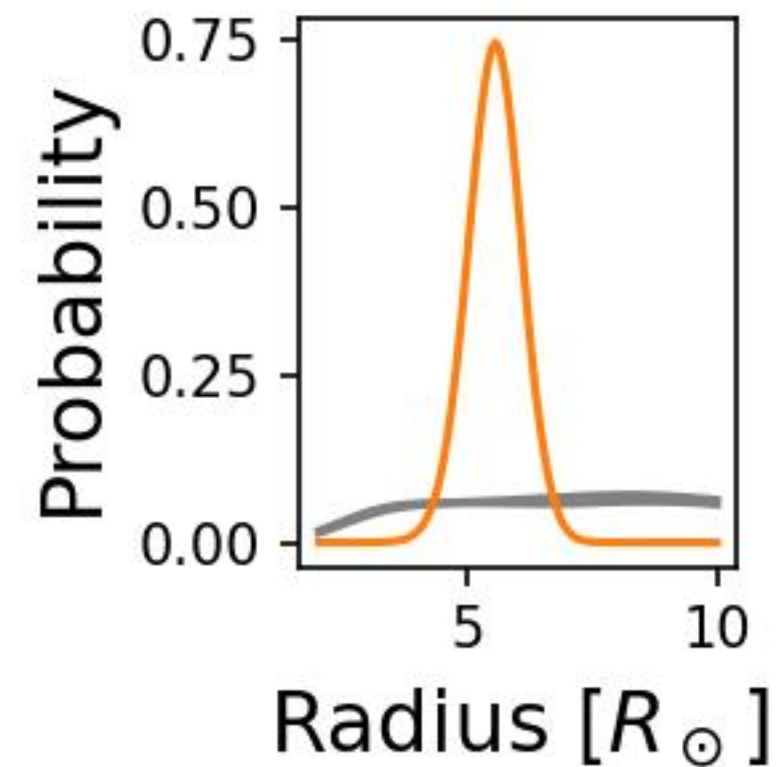
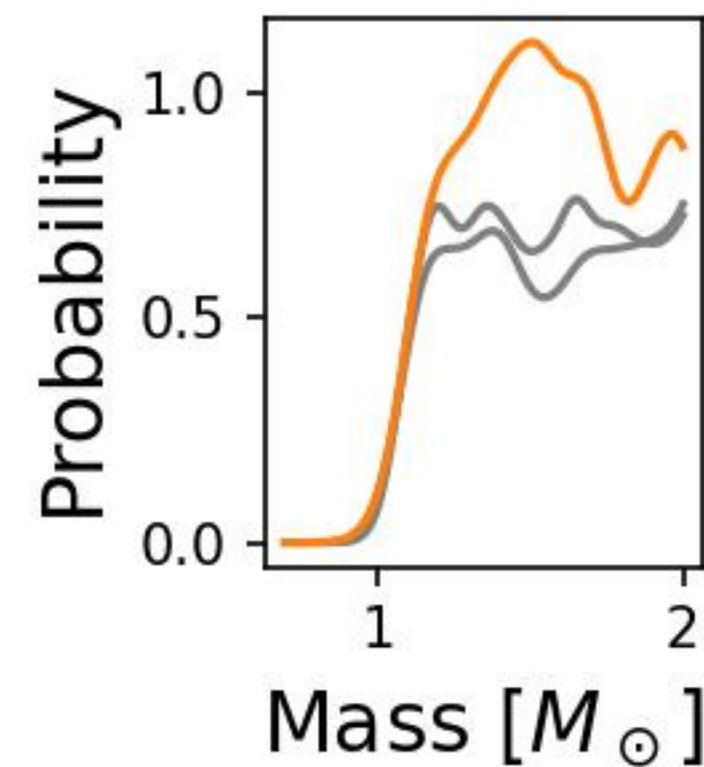
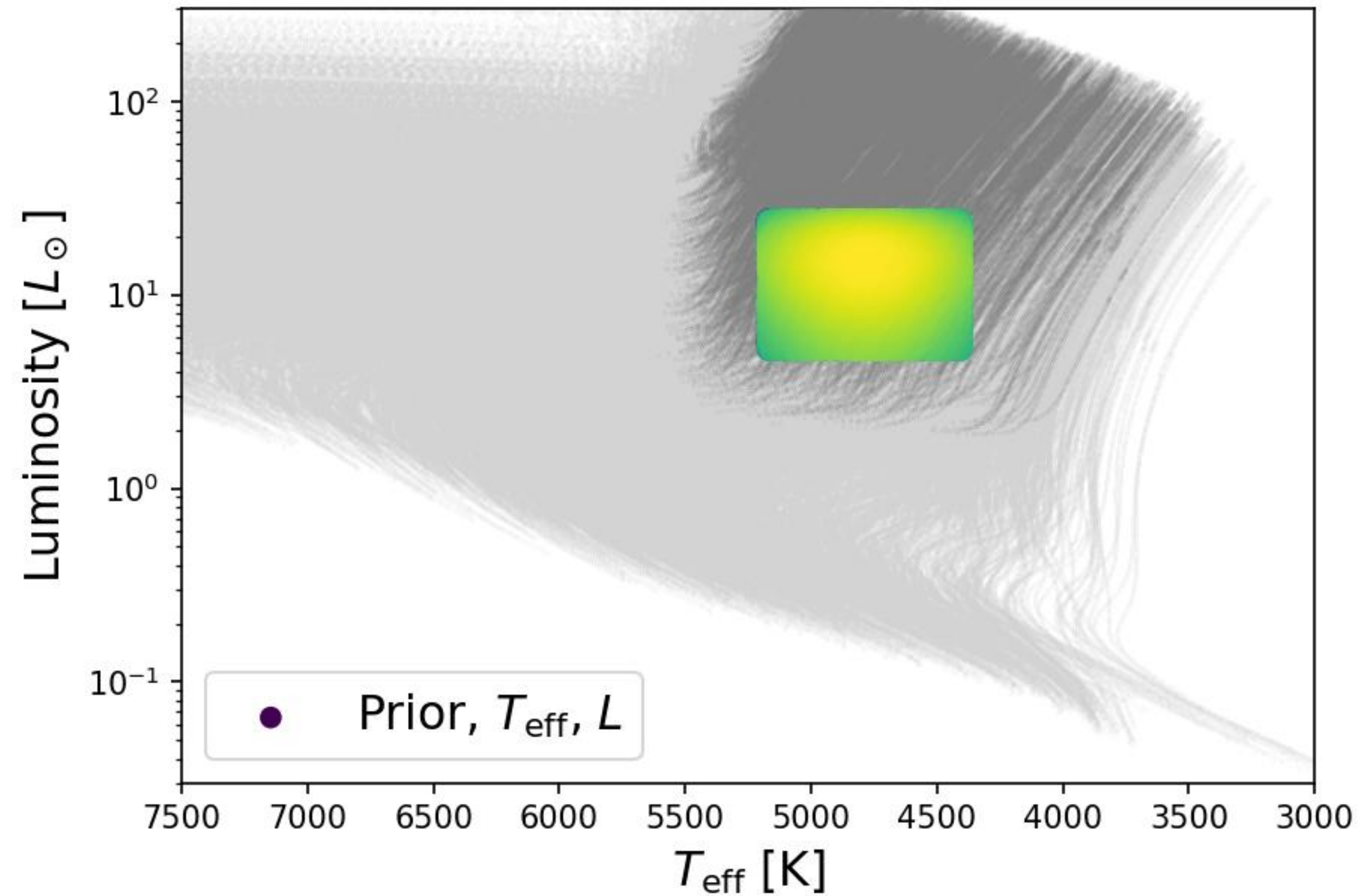
Constraints with asteroseismology

$M, Z, Y, \alpha_{\text{MLT}}, \tau$



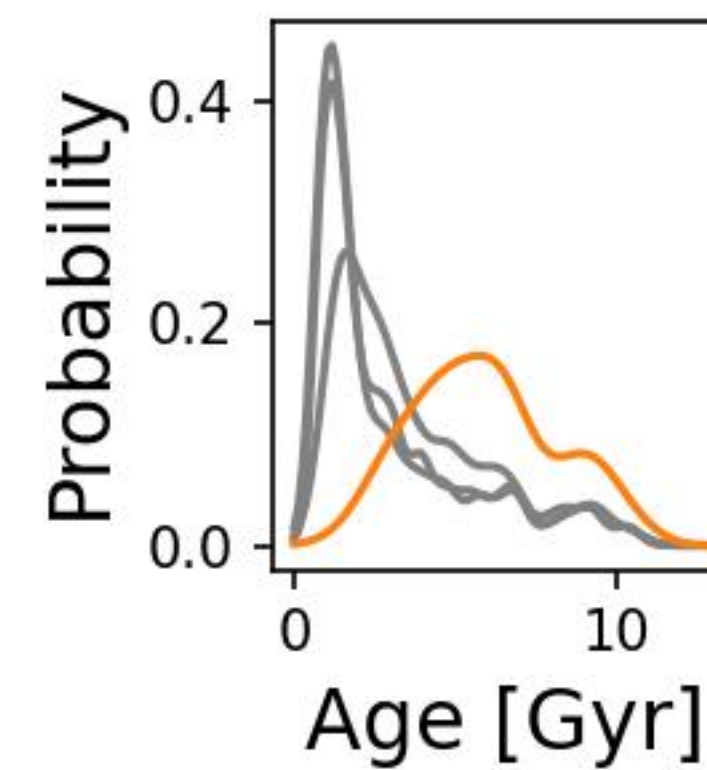
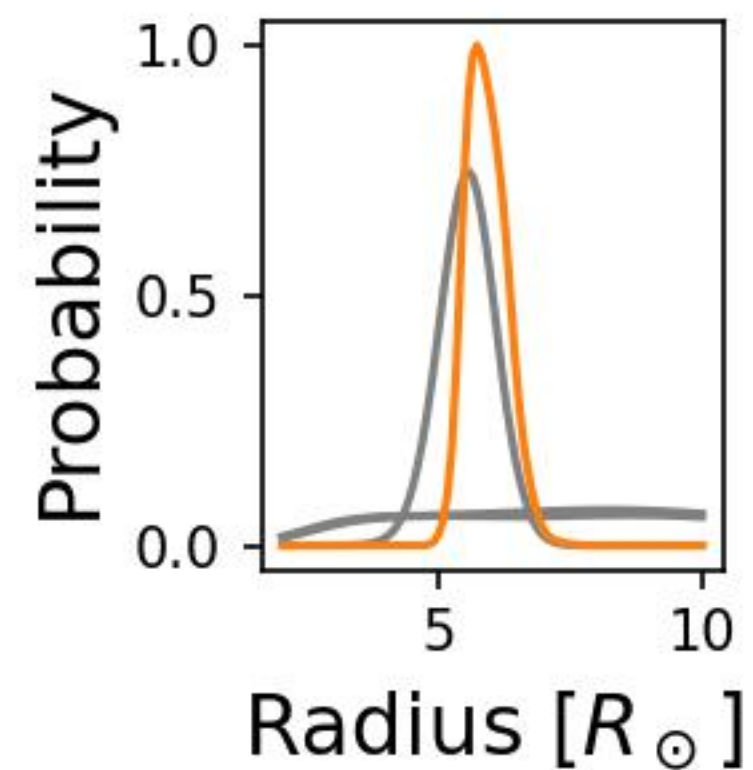
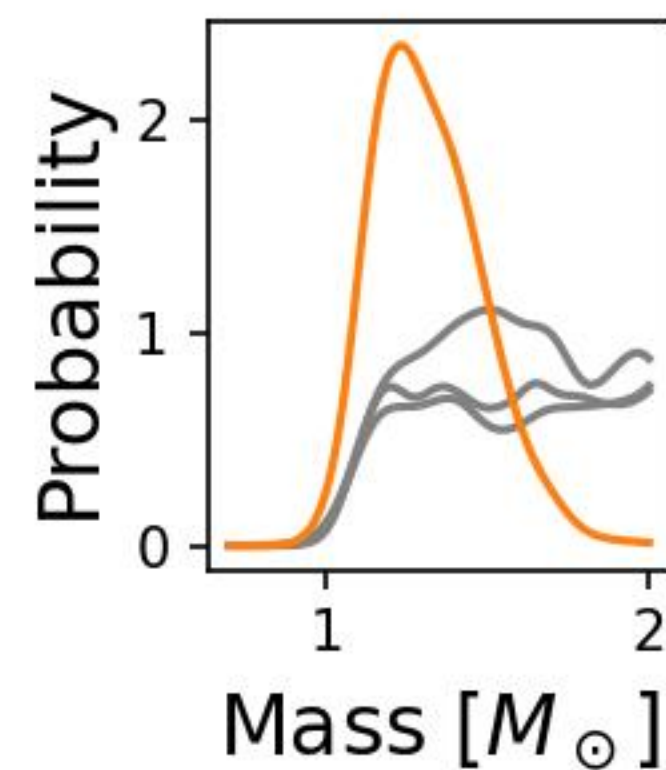
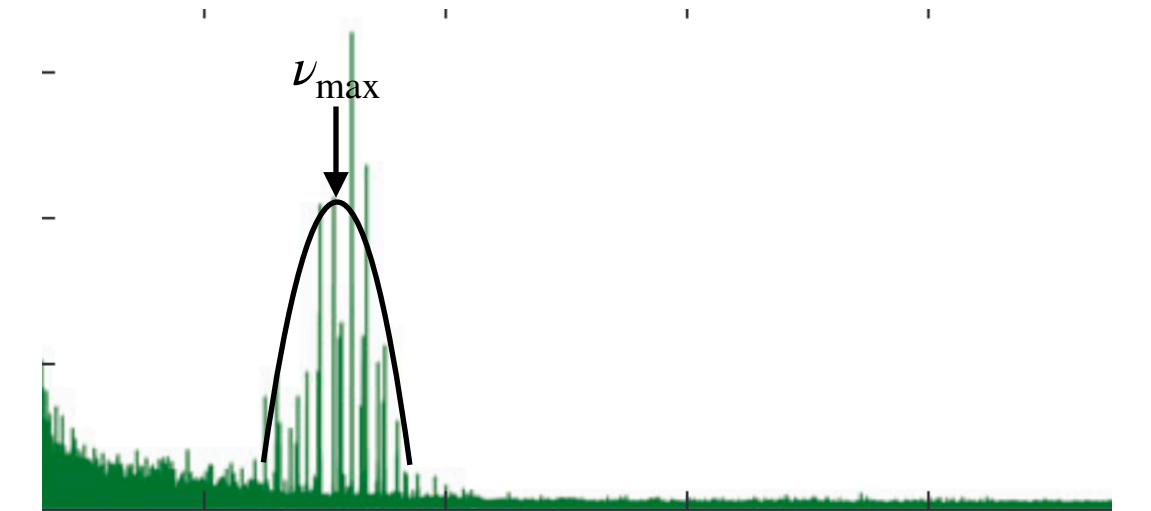
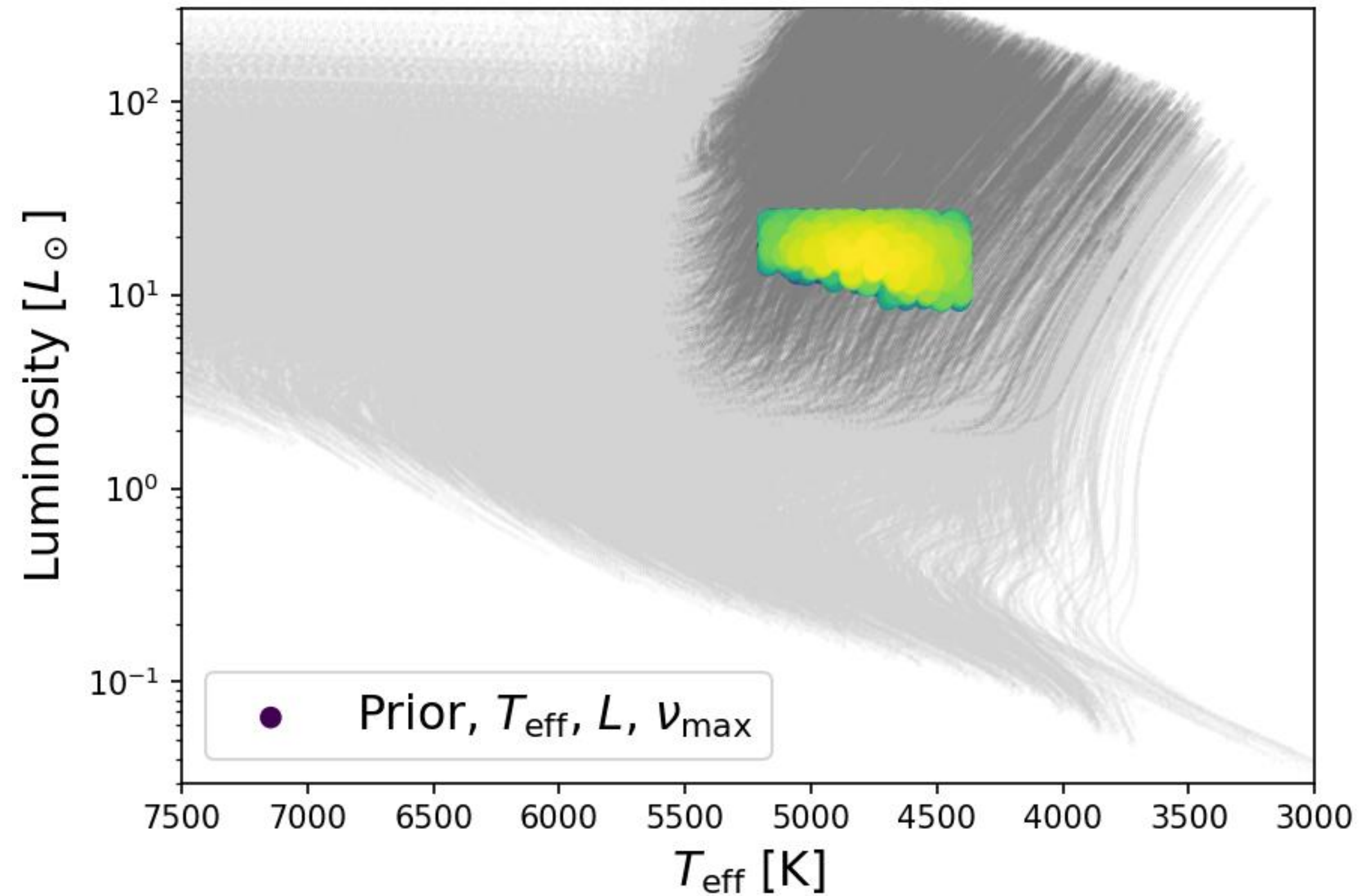
Constraints with asteroseismology

$M, Z, Y, \alpha_{\text{MLT}}, \tau$



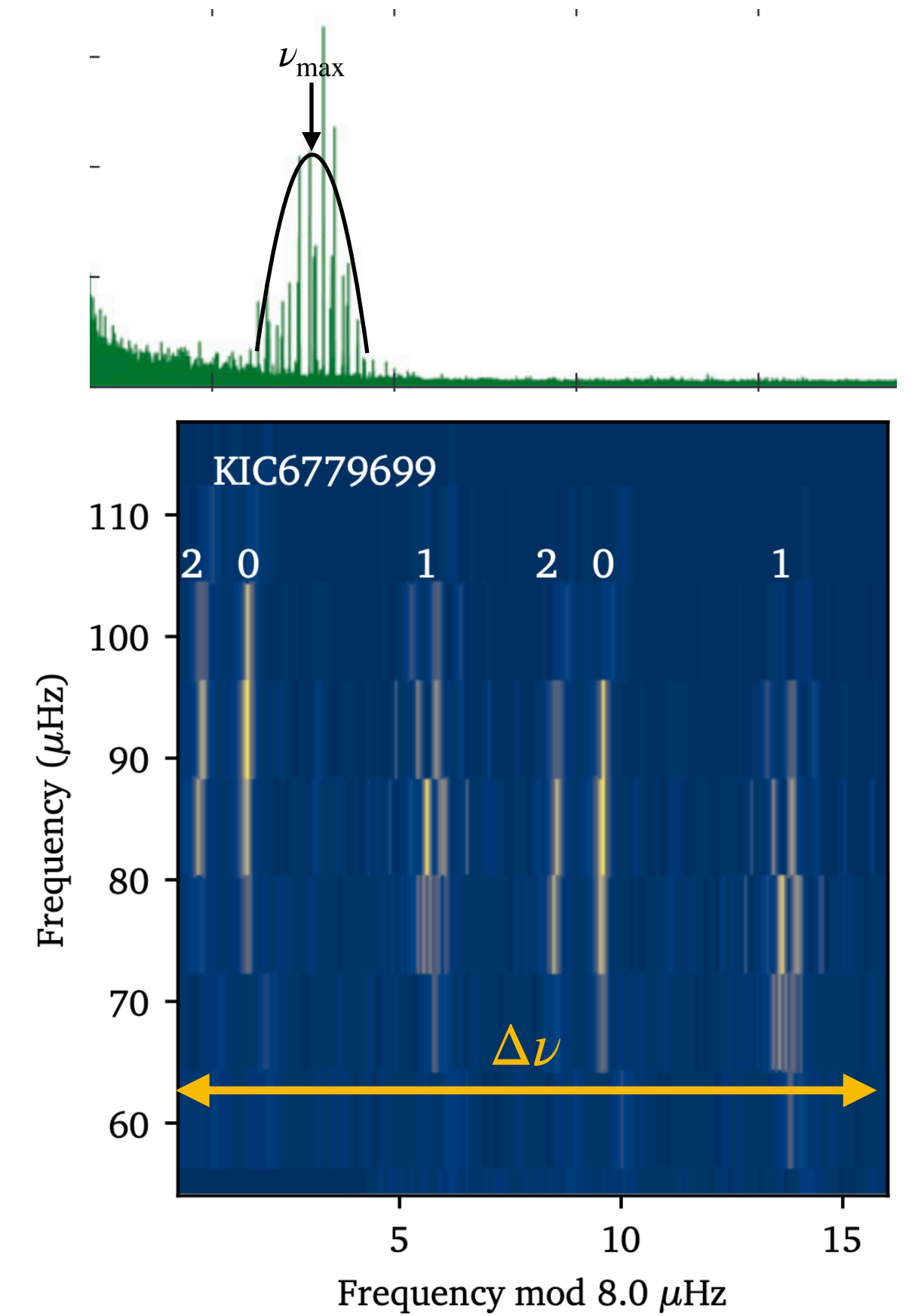
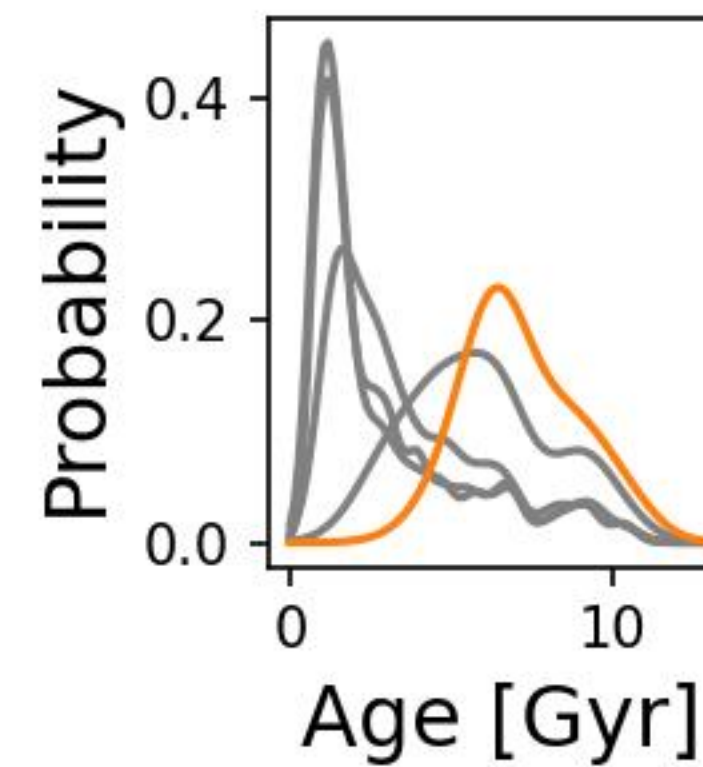
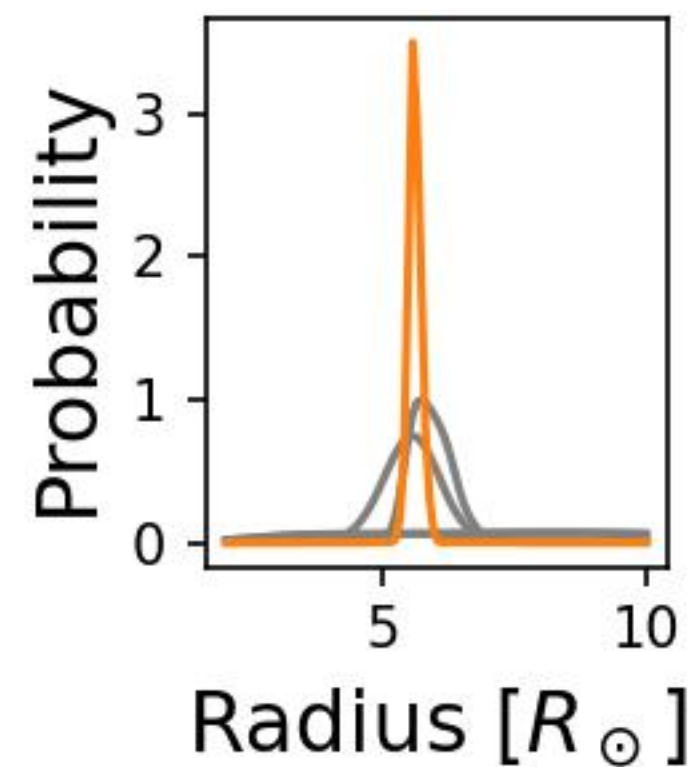
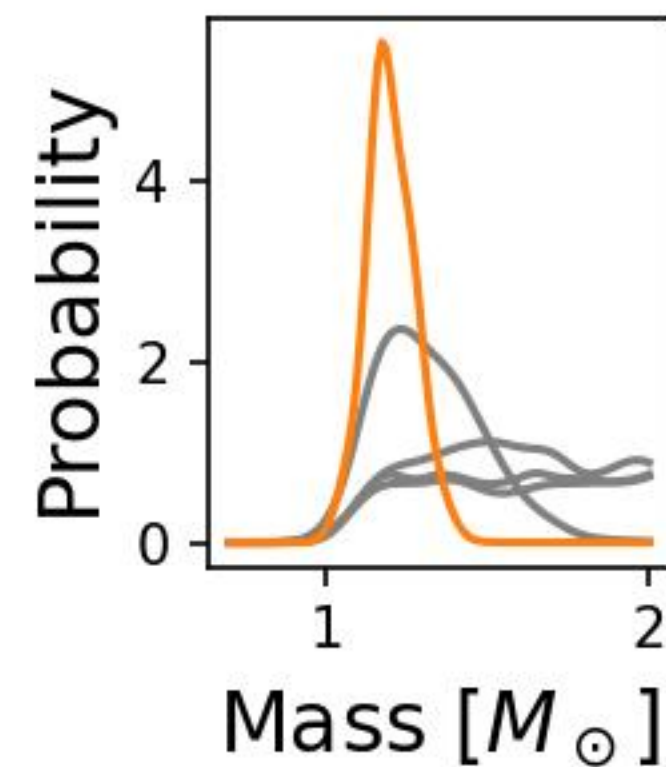
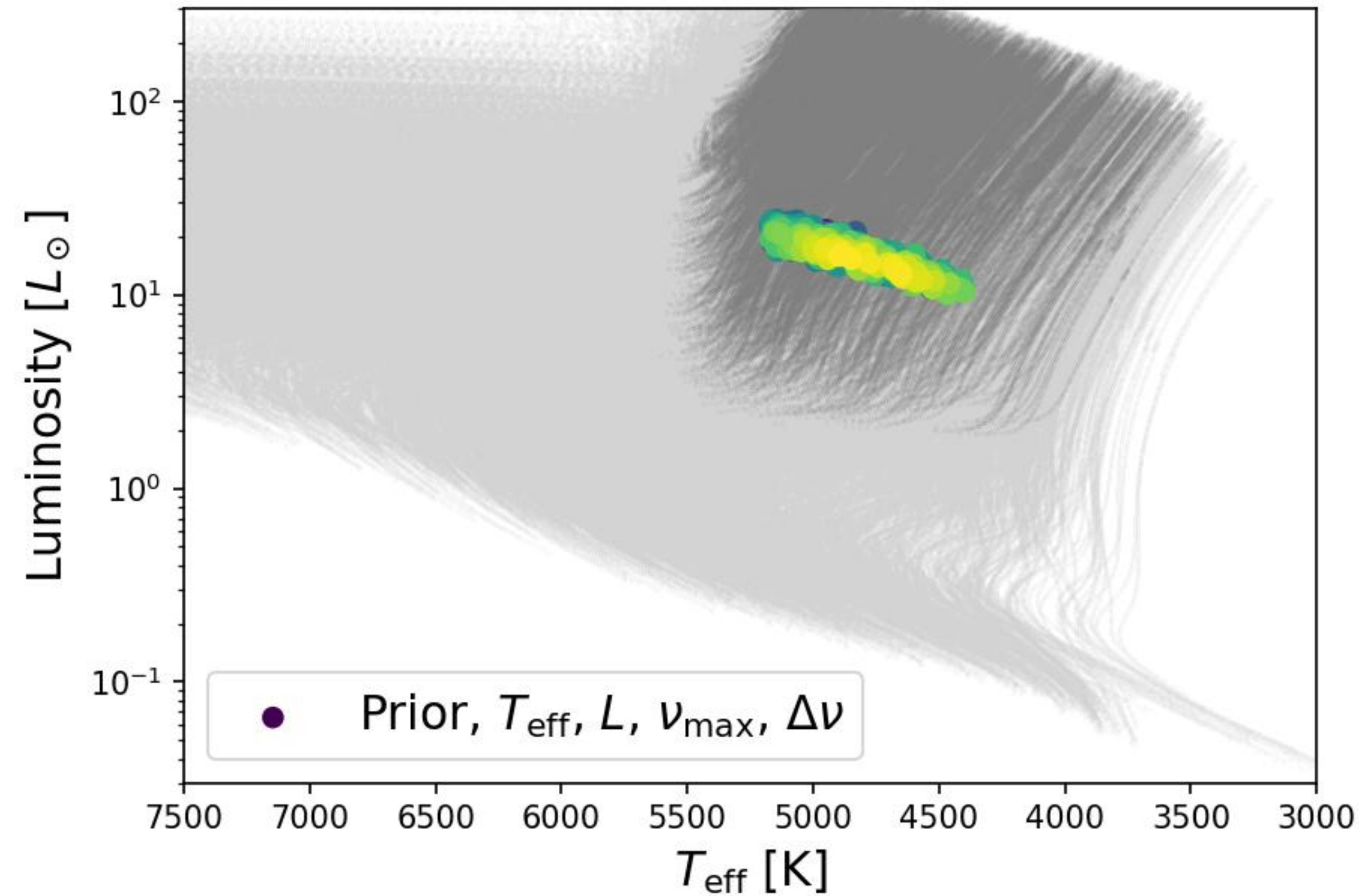
Constraints with asteroseismology

$M, Z, Y, \alpha_{\text{MLT}}, \tau$



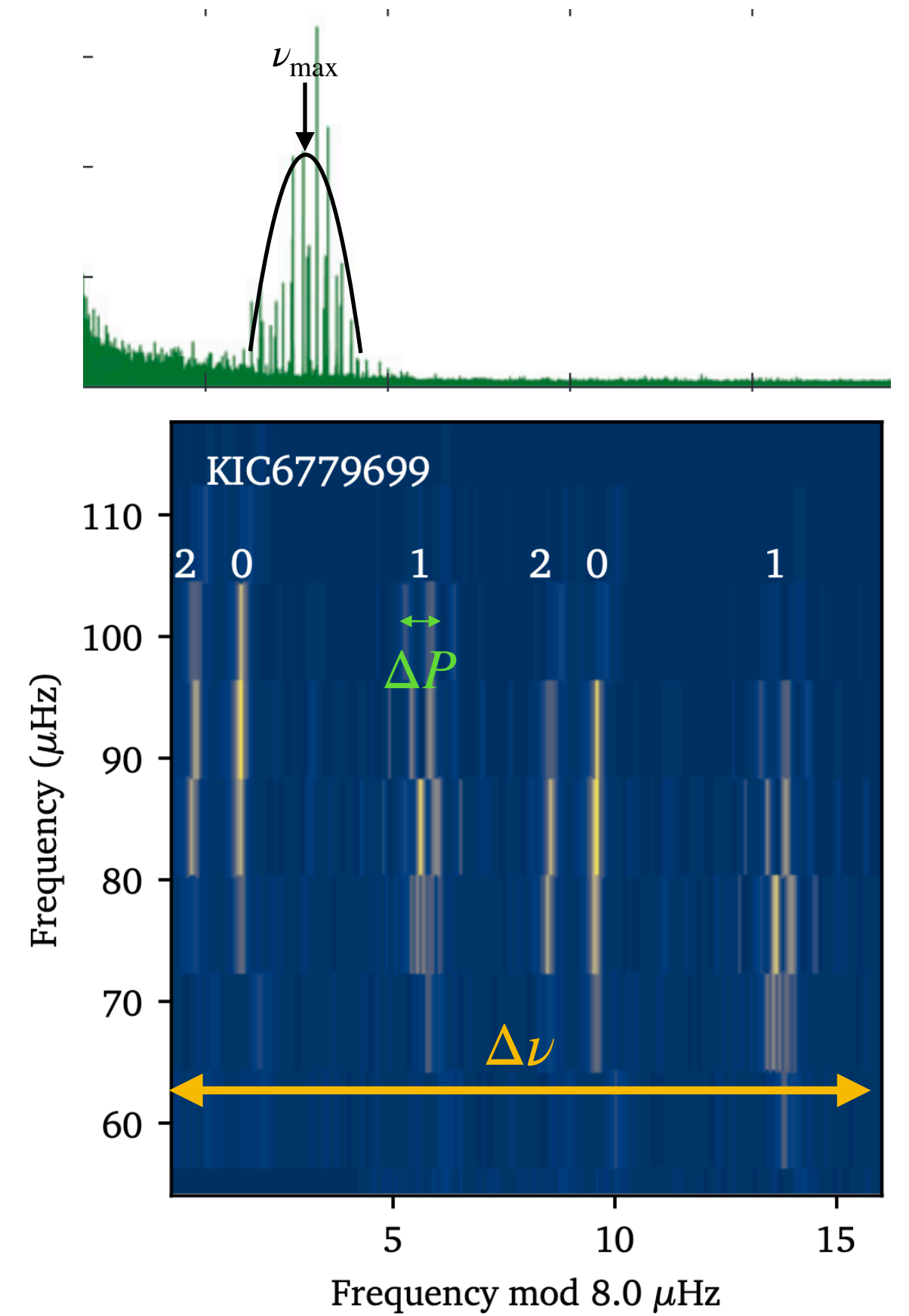
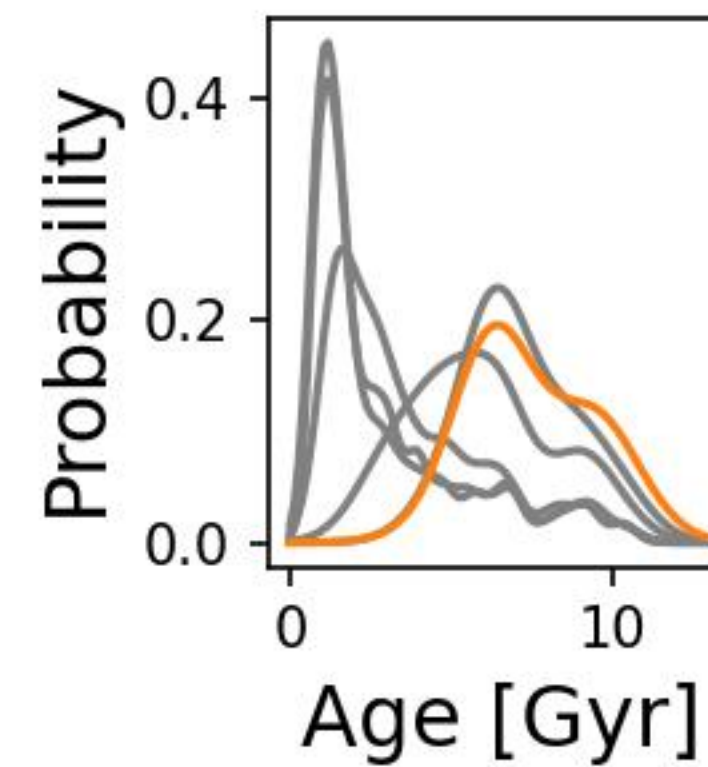
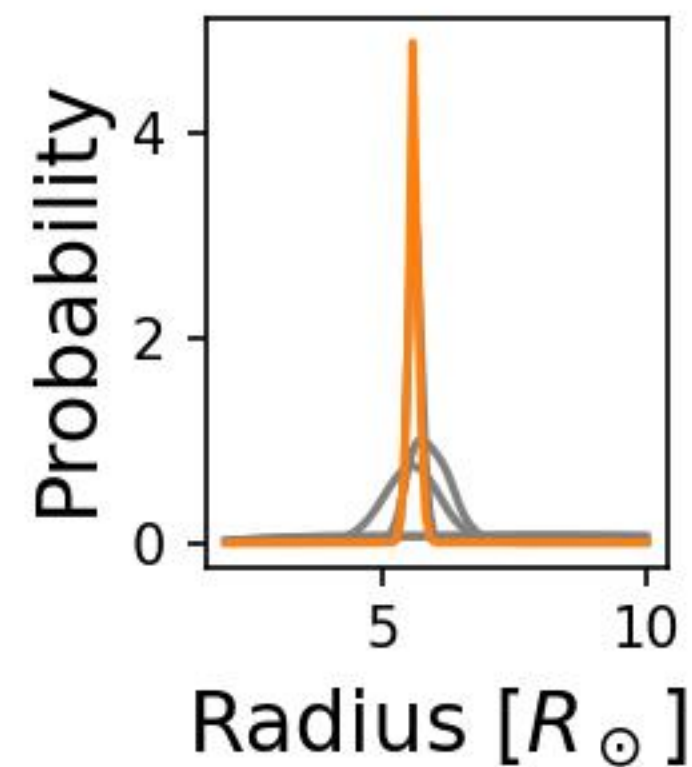
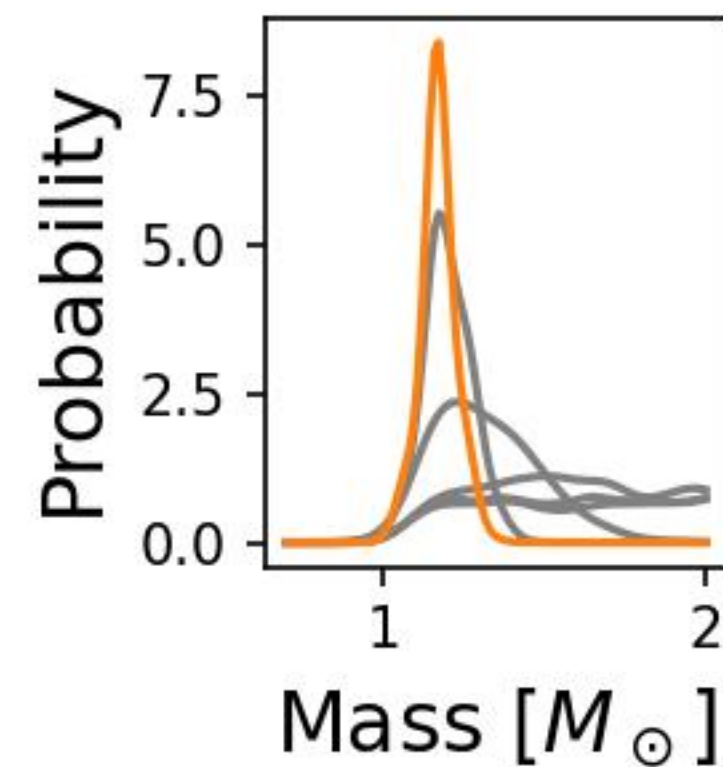
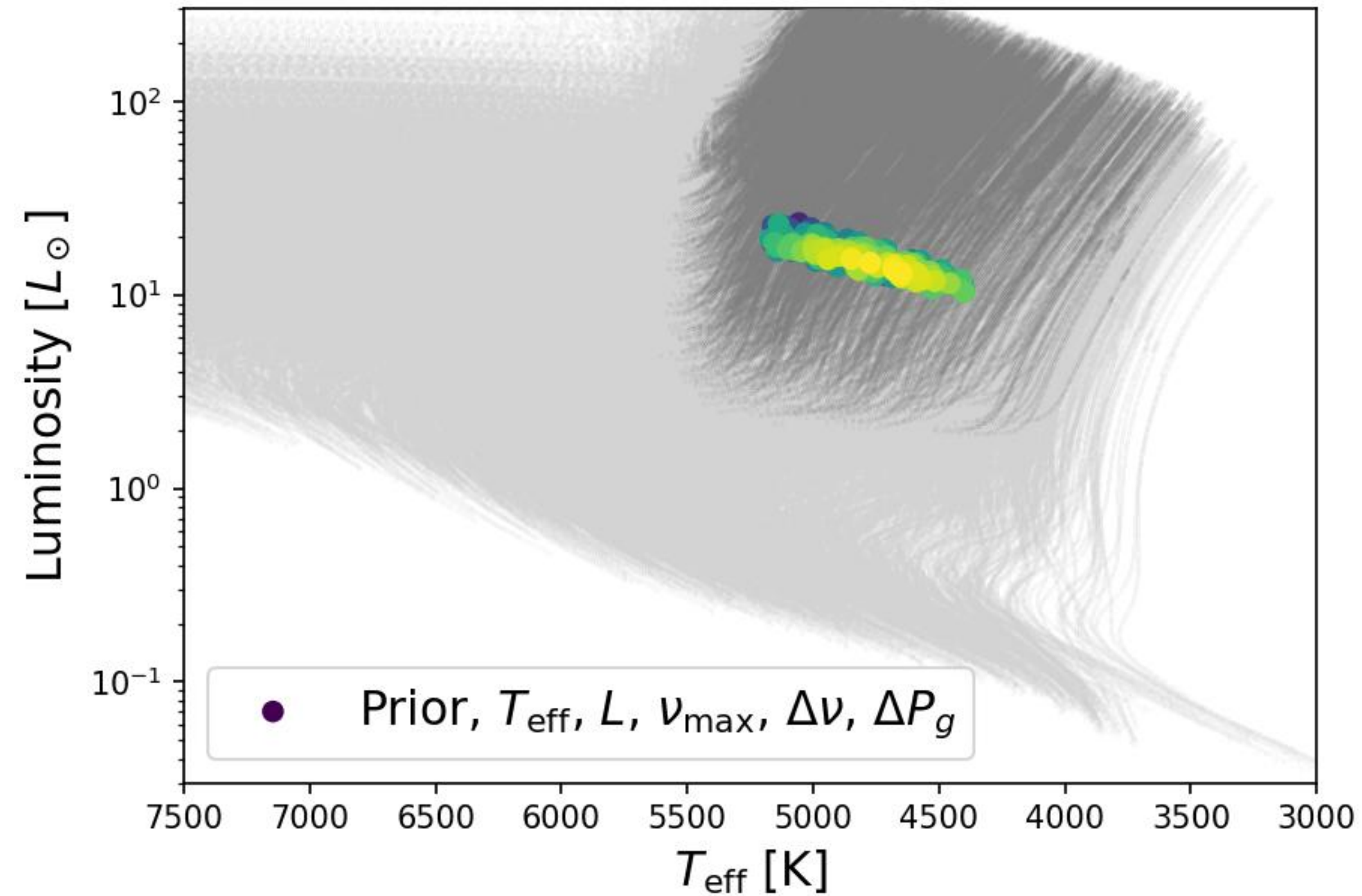
Constraints with asteroseismology

$M, Z, Y, \alpha_{\text{MLT}}, \tau$

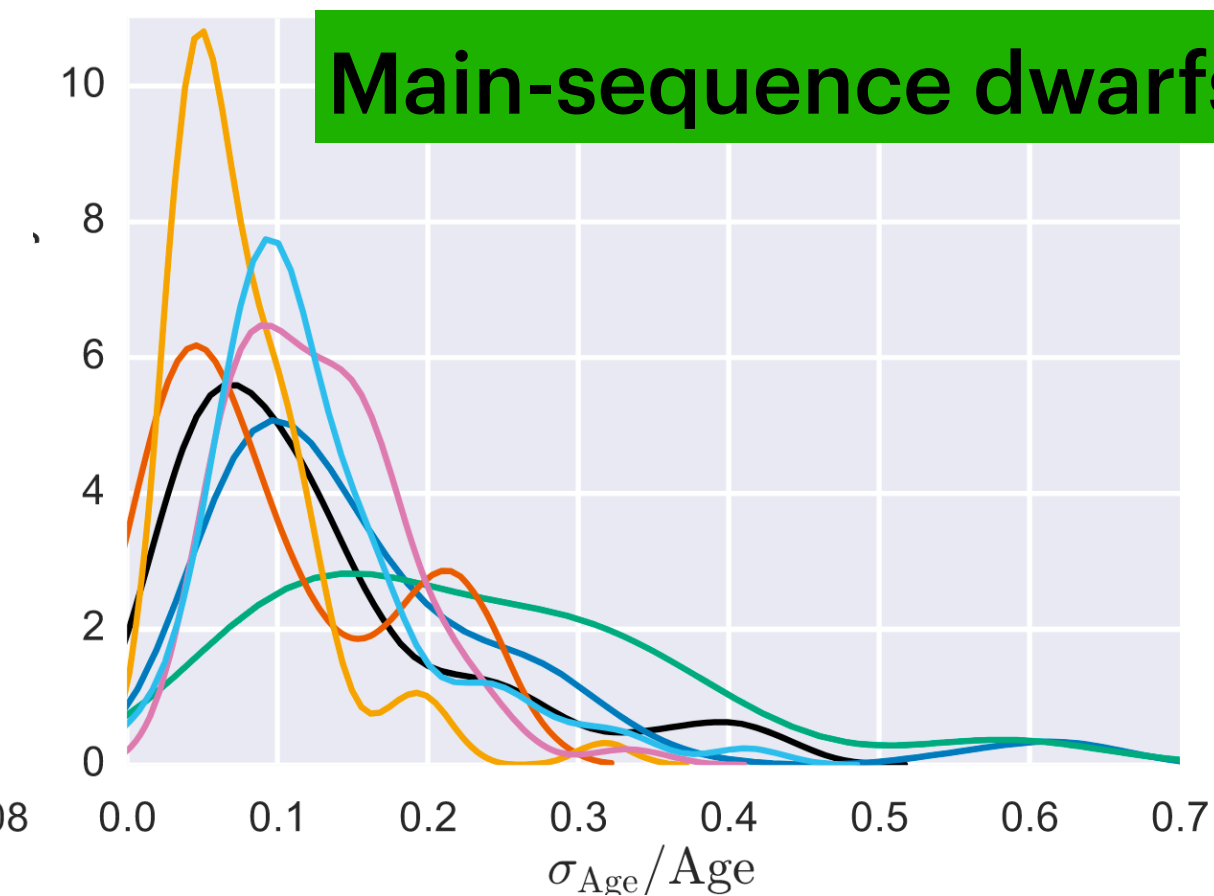
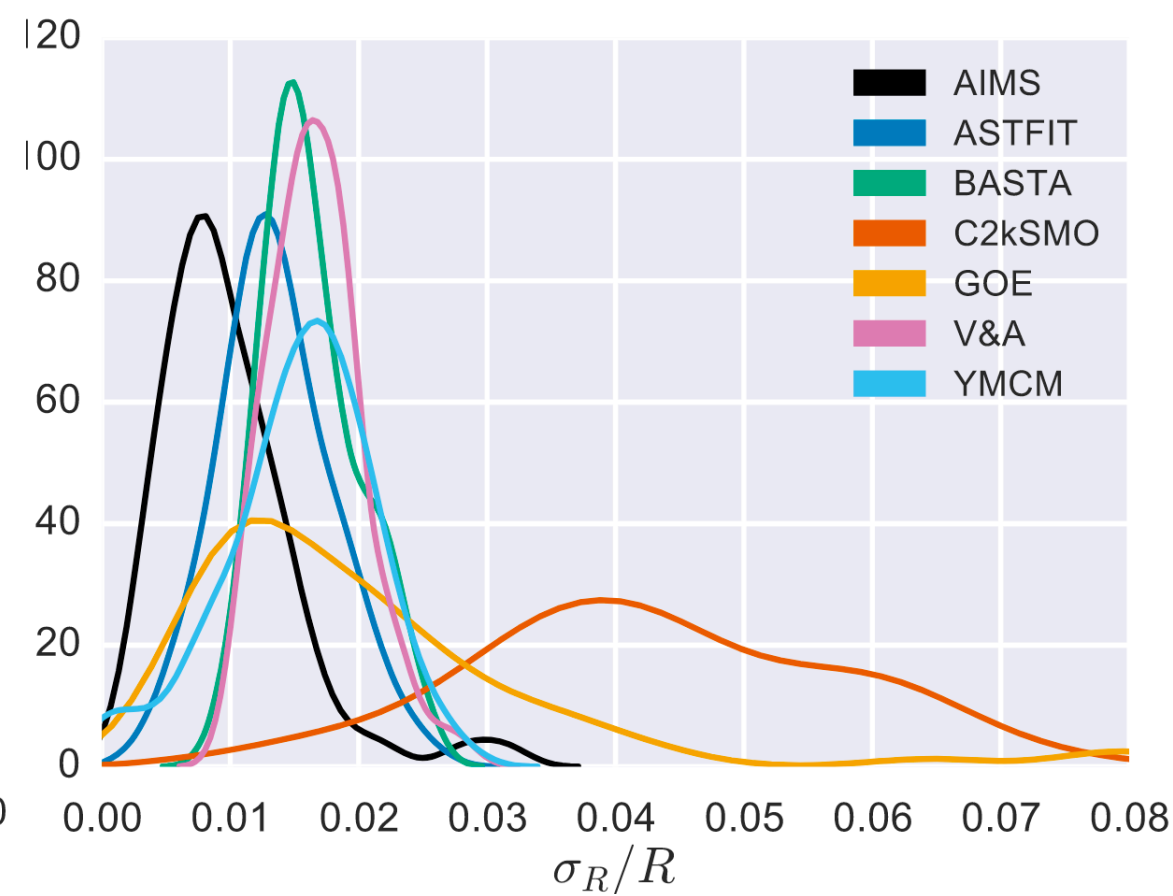
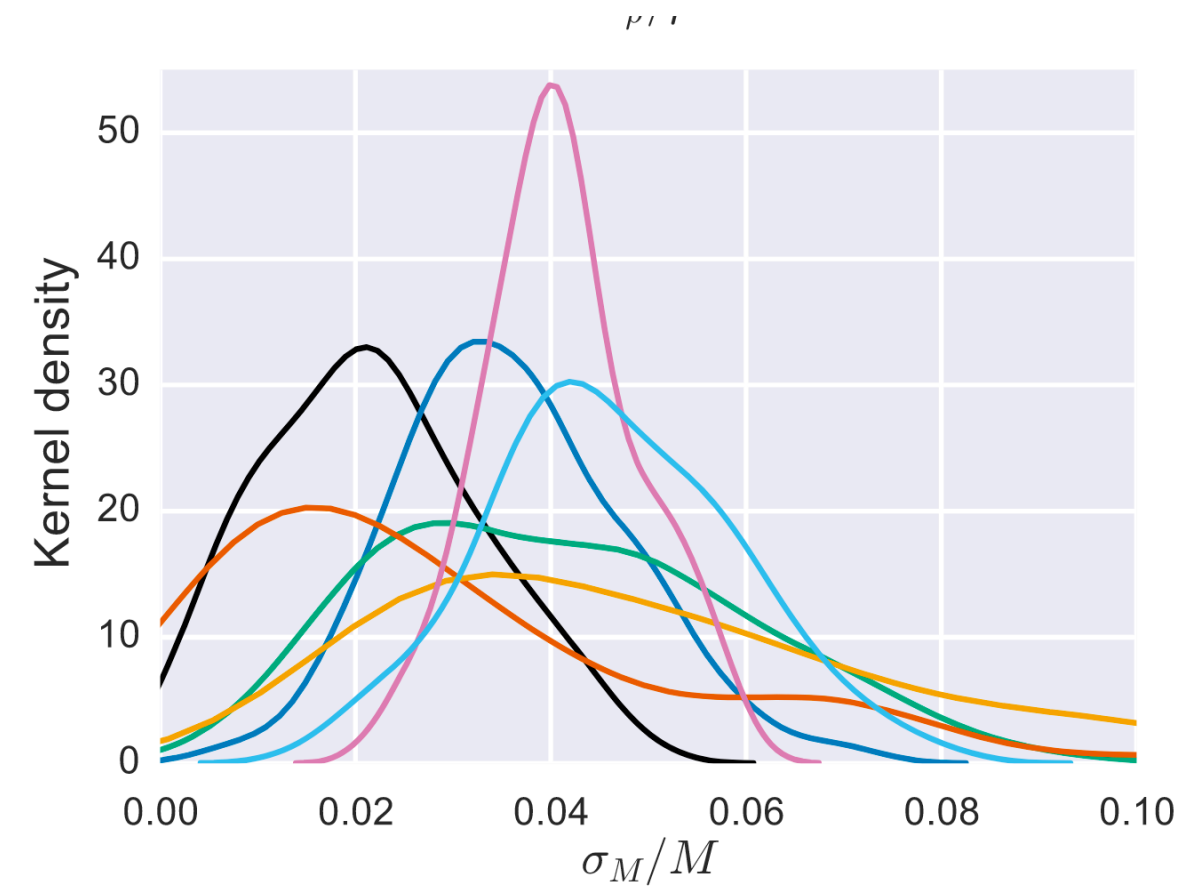
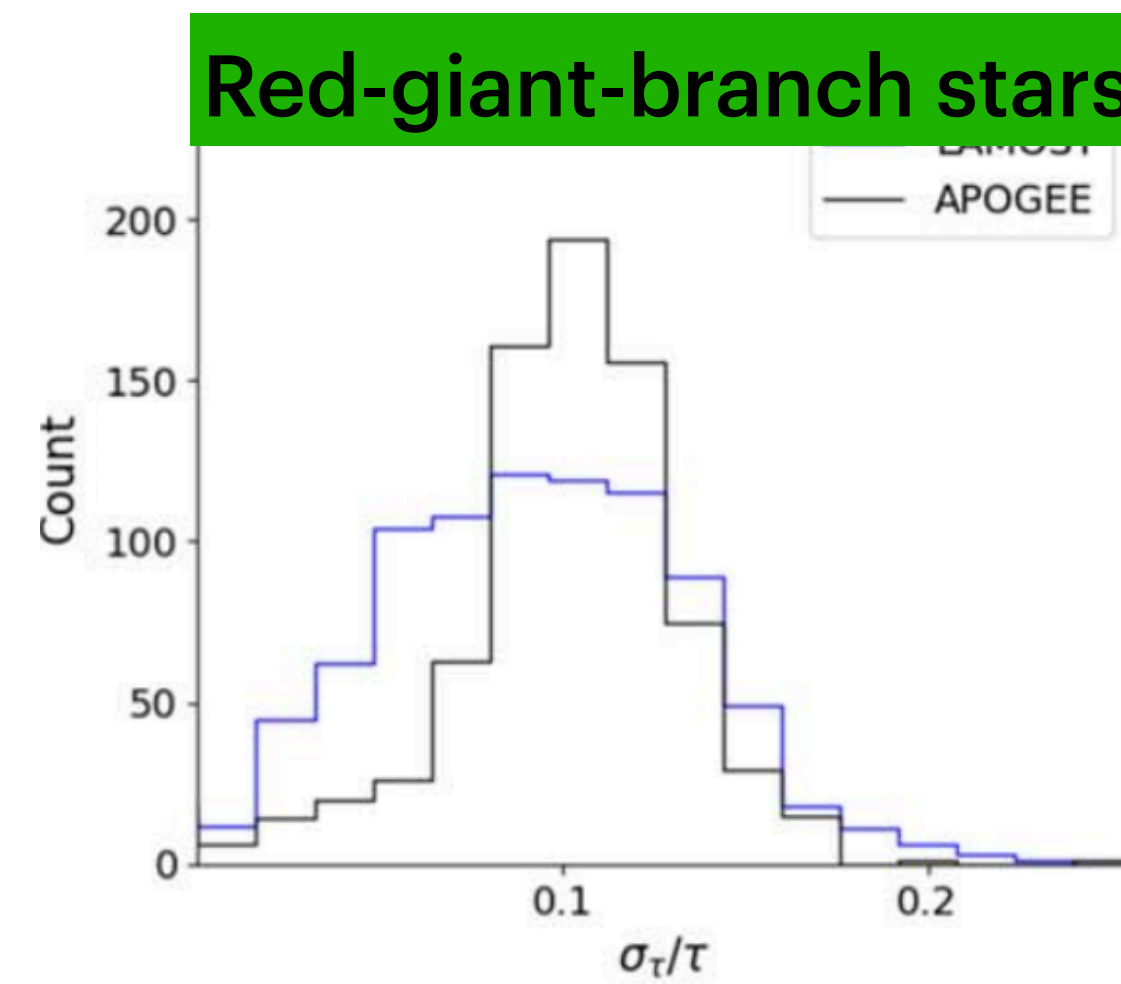
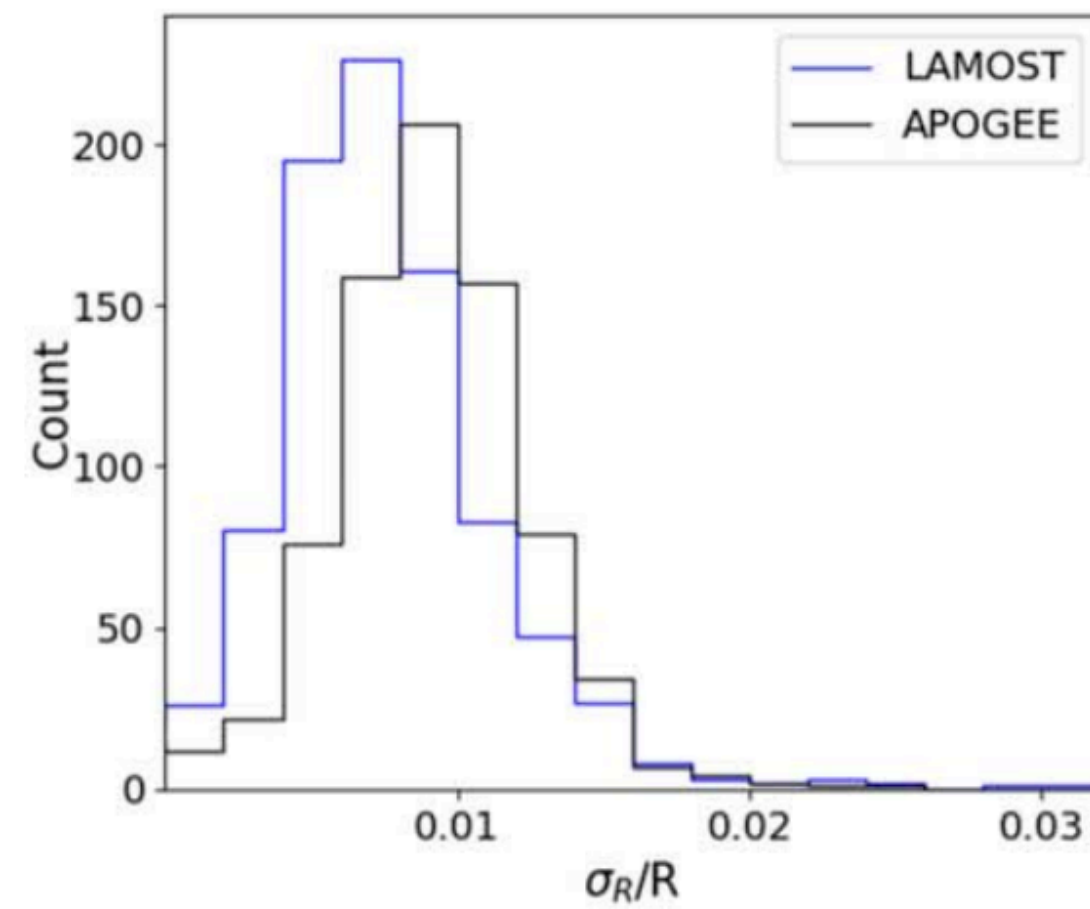
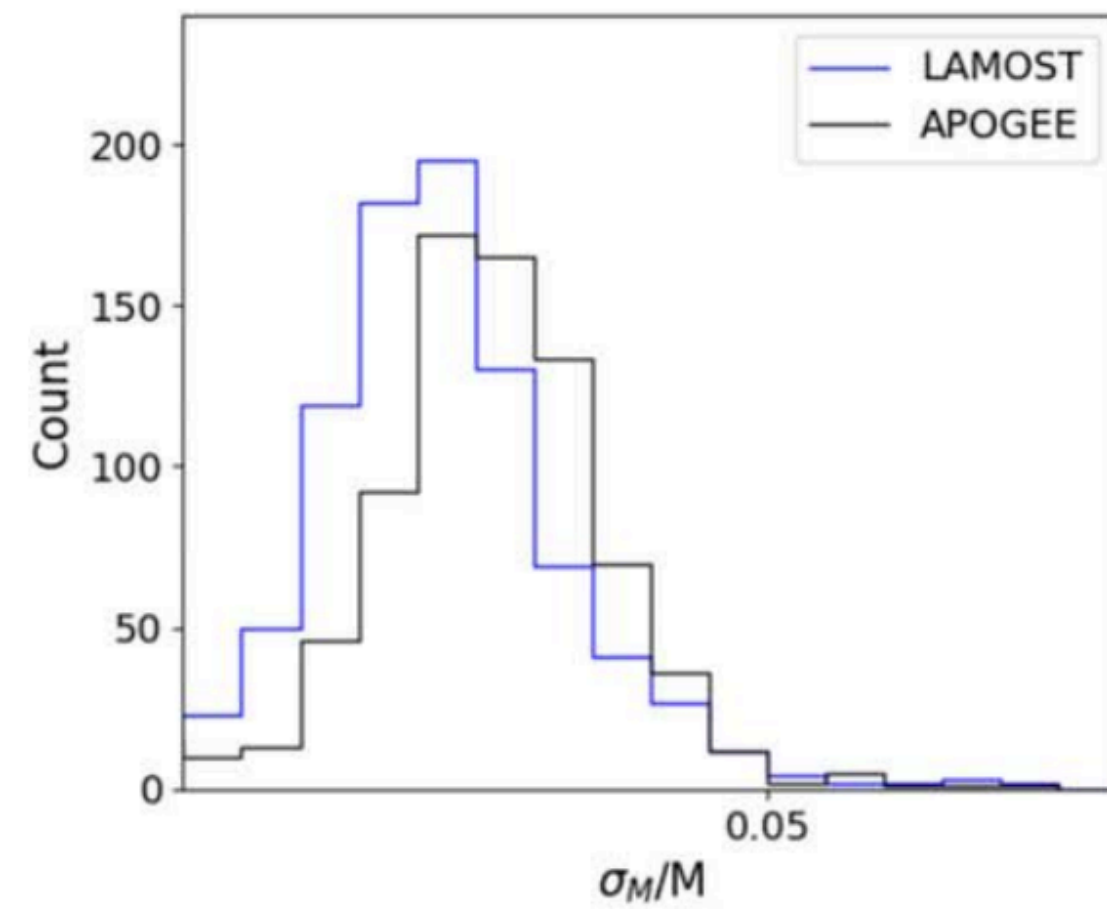


Constraints with asteroseismology

$M, Z, Y, \alpha_{\text{MLT}}, \tau$



Typical Uncertainties



Montalban et al. (2021)

Li, Tanda et al. (2022)

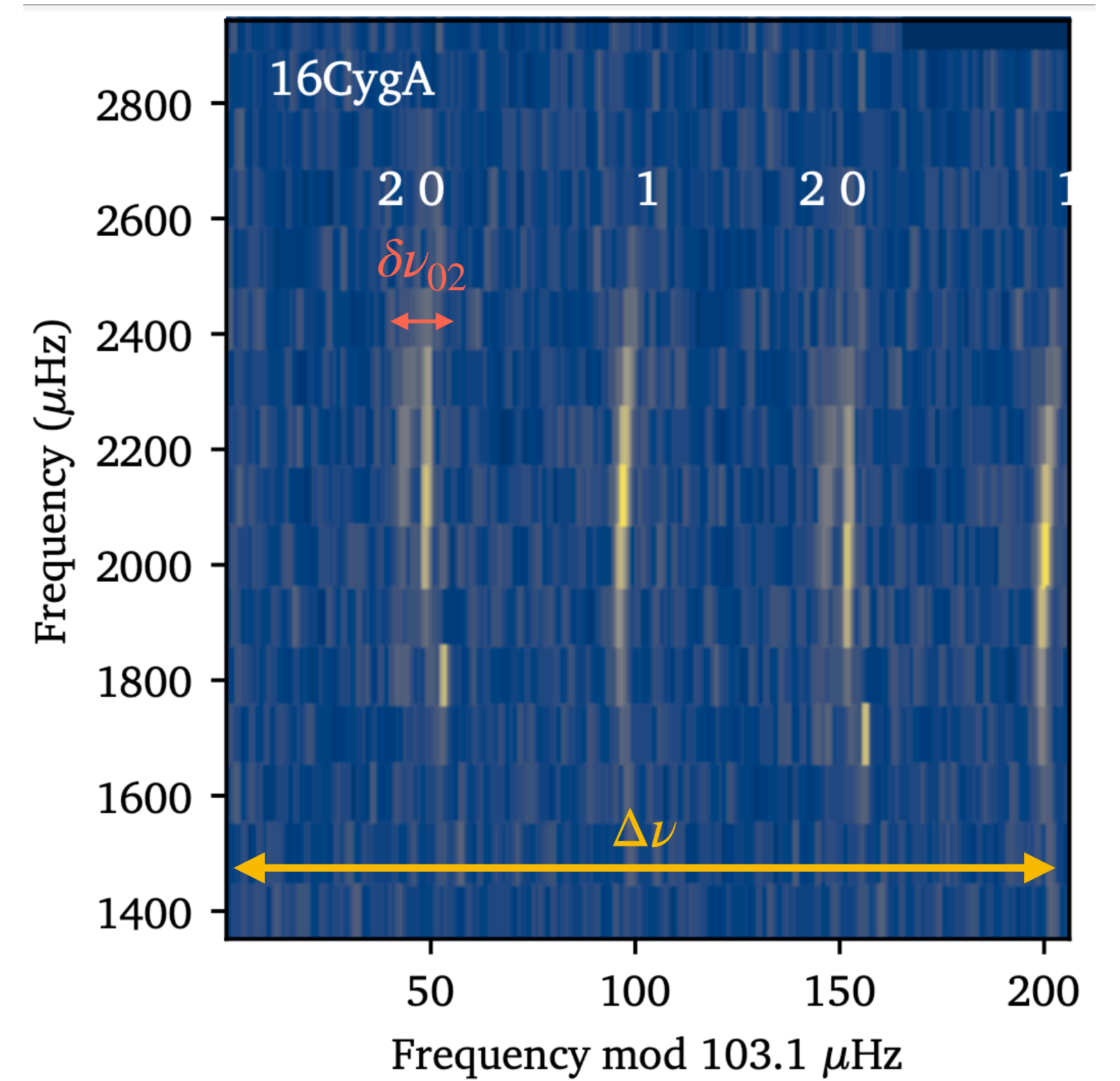
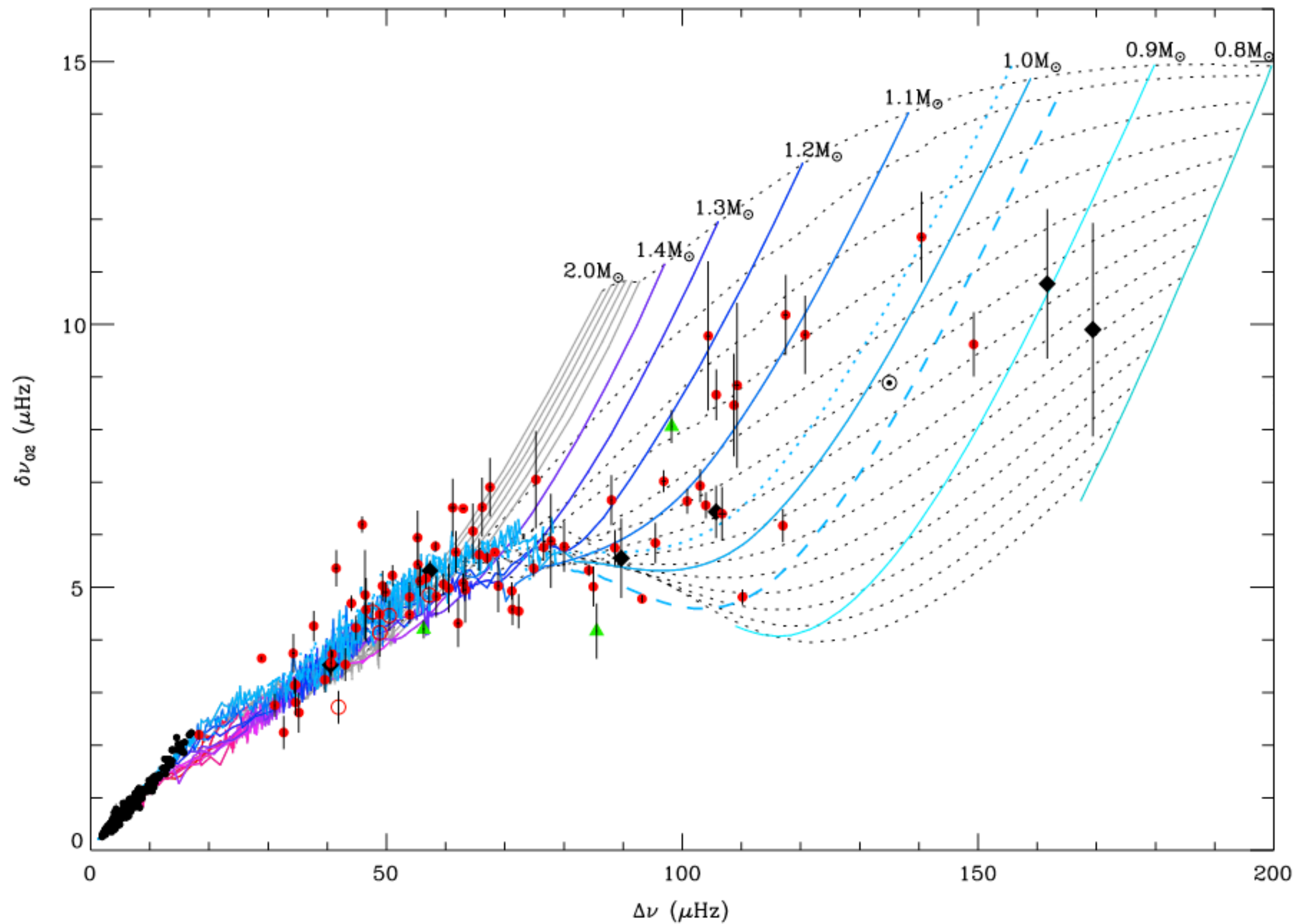
* Wang, Yingxiang et al. (2023)

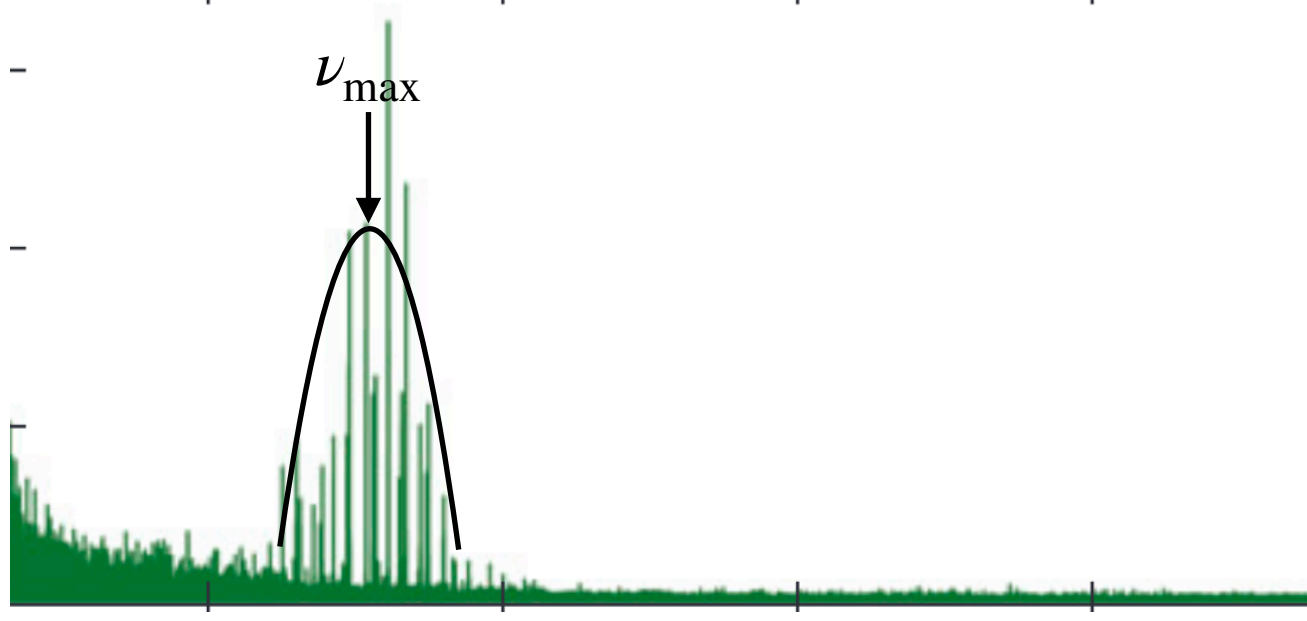
* Silva Aguirre et al. (2017)

Hall et al. (2021)

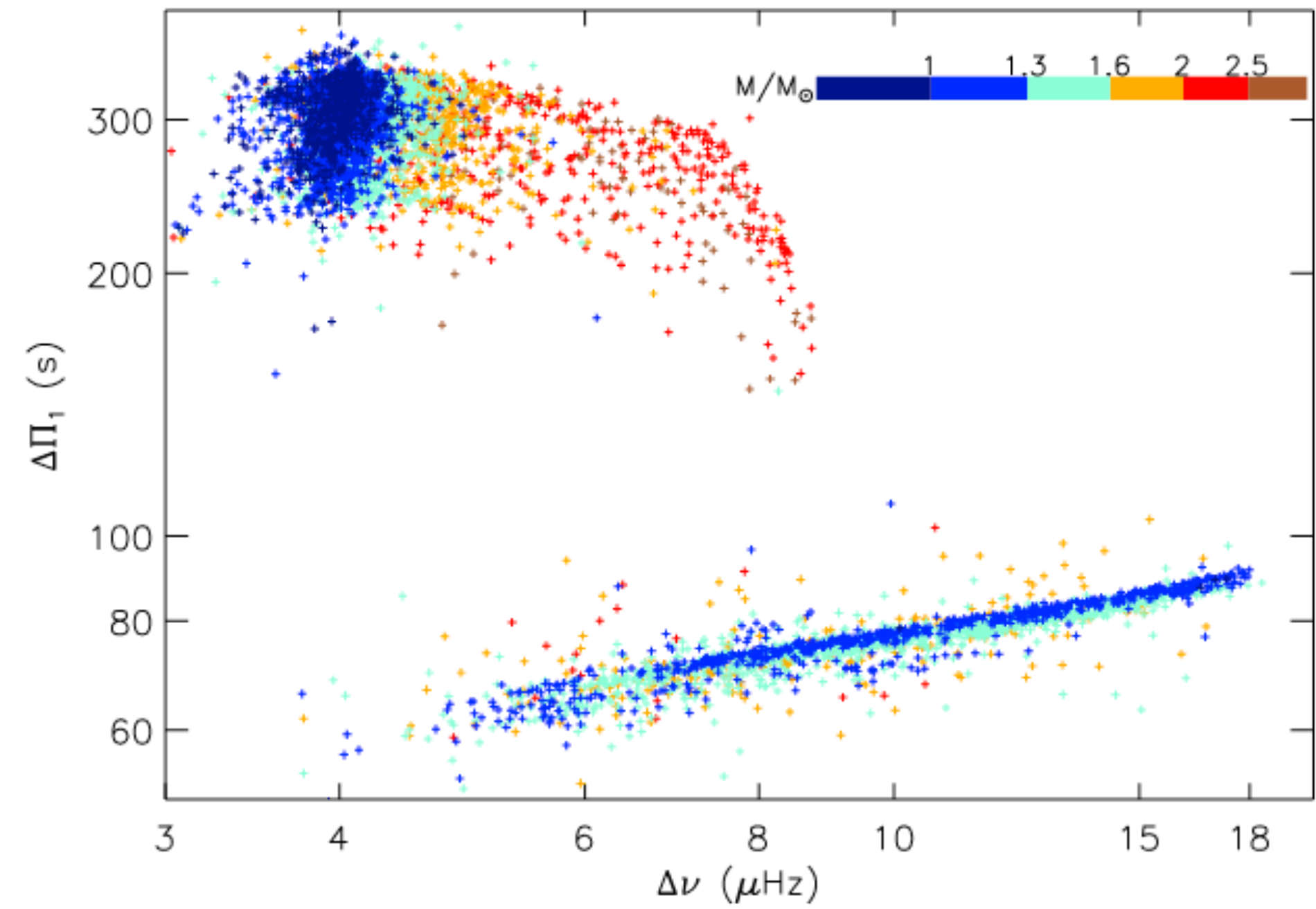
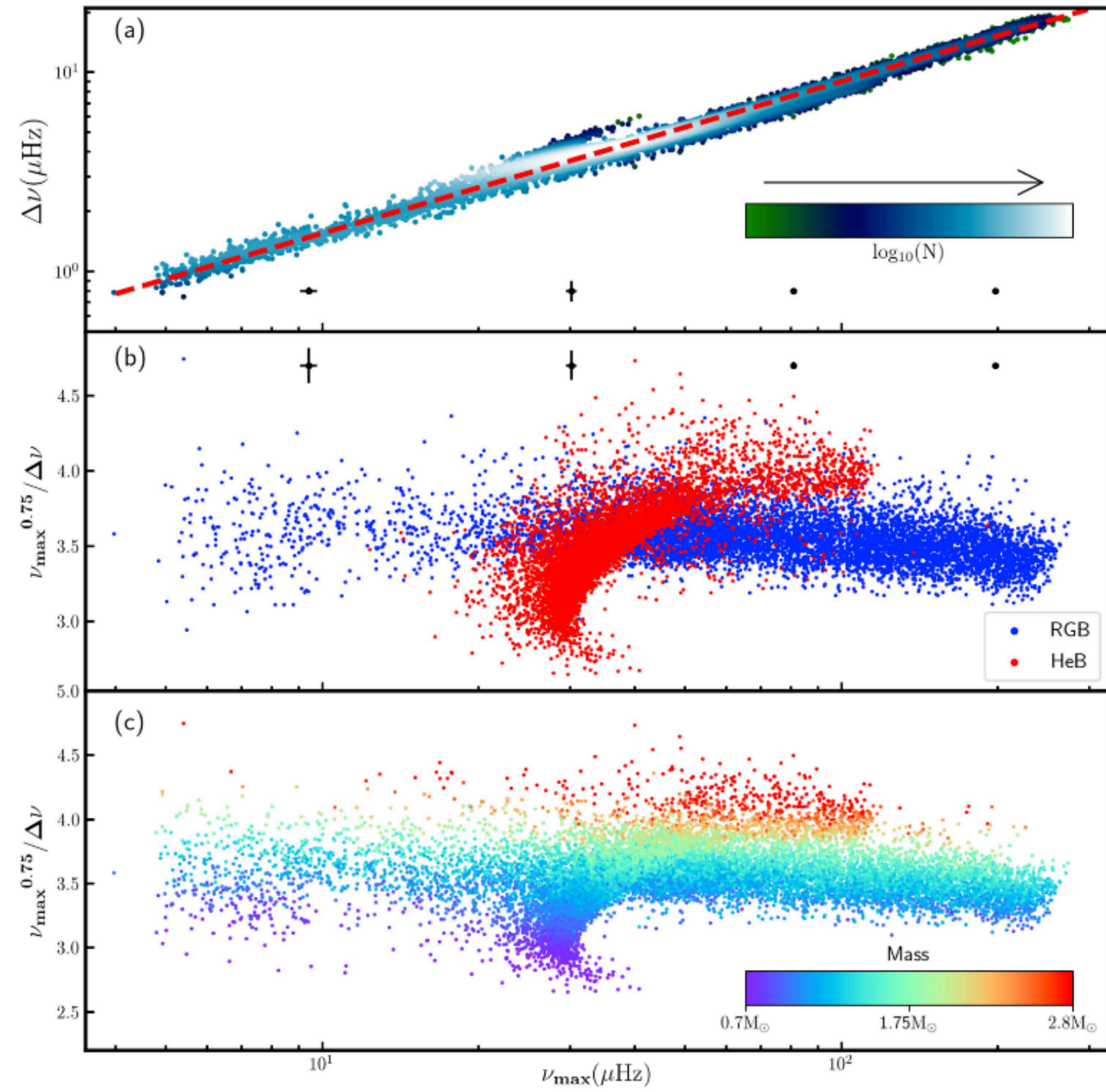
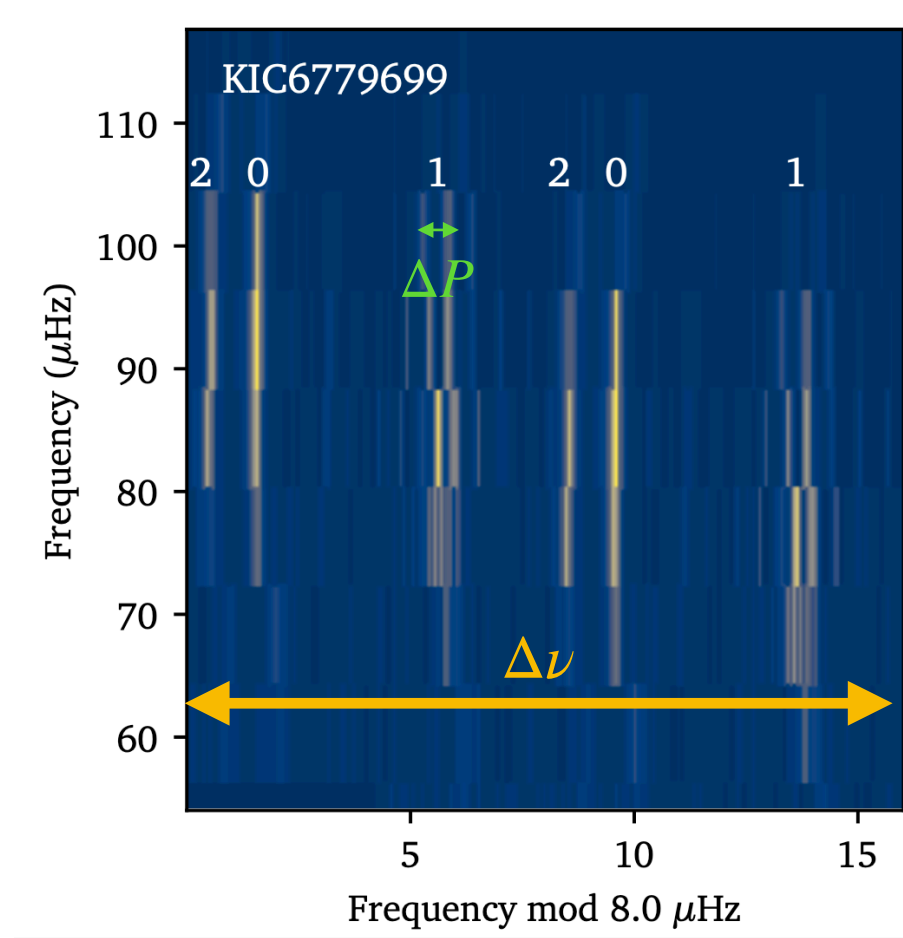
Main-sequence Stars

C-D diagram





Red Giants



Fitting Fundamentals

Bayesian Framework

- Data (stellar observables L , $[M/H]$, T_{eff} , ν_{max} , $\{\nu_{n,l}\}$)
- Model (stellar models, i.e. stellar properties L , $[M/H]$, T_{eff} , ν_{max} , $\{\nu_{n,l}\}$, M_{core} , M , t , $\dots$ at certain M_{init} , Y_{init} , Z_{init} , t)
- Bayesian theorem
 - $p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)}$
 - Posterior = $\frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}$
- We wish to do proper sampling with MCMC
 - This would require an interpolator.
 - In reality, we rely on pre-computed models (samples are fixed)
- $p_i \propto w_i L_i$

Priors

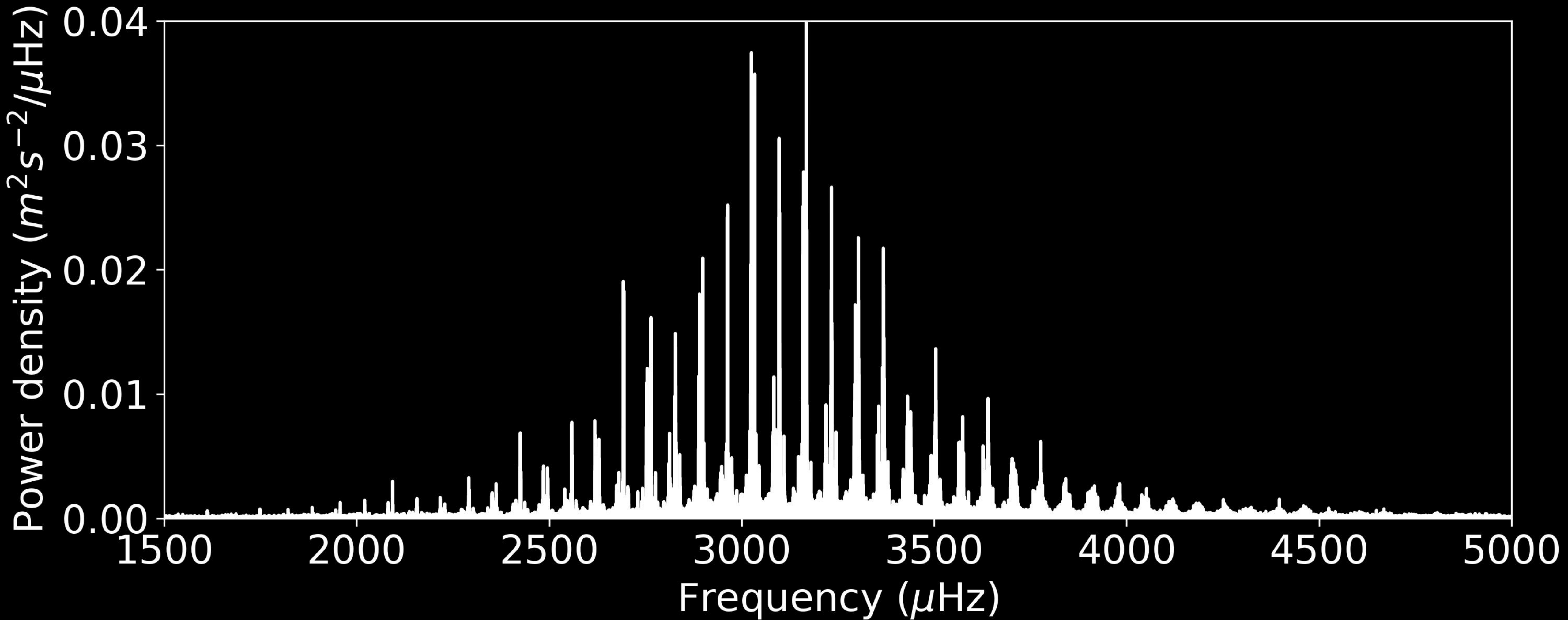
$$p(\theta | D, M) = \frac{p(\theta | M) p(D | \theta, M)}{p(D | M)}$$

- Initial mass function
 - $p(M) = \text{IMF}(M)$
- Numerical time step is not proportional to the rate of stellar evolution
 - $p(\Delta t) \propto \Delta t$
- Coeval systems (binaries and stellar clusters)
 - $p(\tau_1, \tau_2) \propto (\tau_1 - \tau_2)^2 / (2\sigma_\tau^2)$
 - $p(Y_1, Y_2) \propto (Y_1 - Y_2)^2 / (2\sigma_Y^2)$

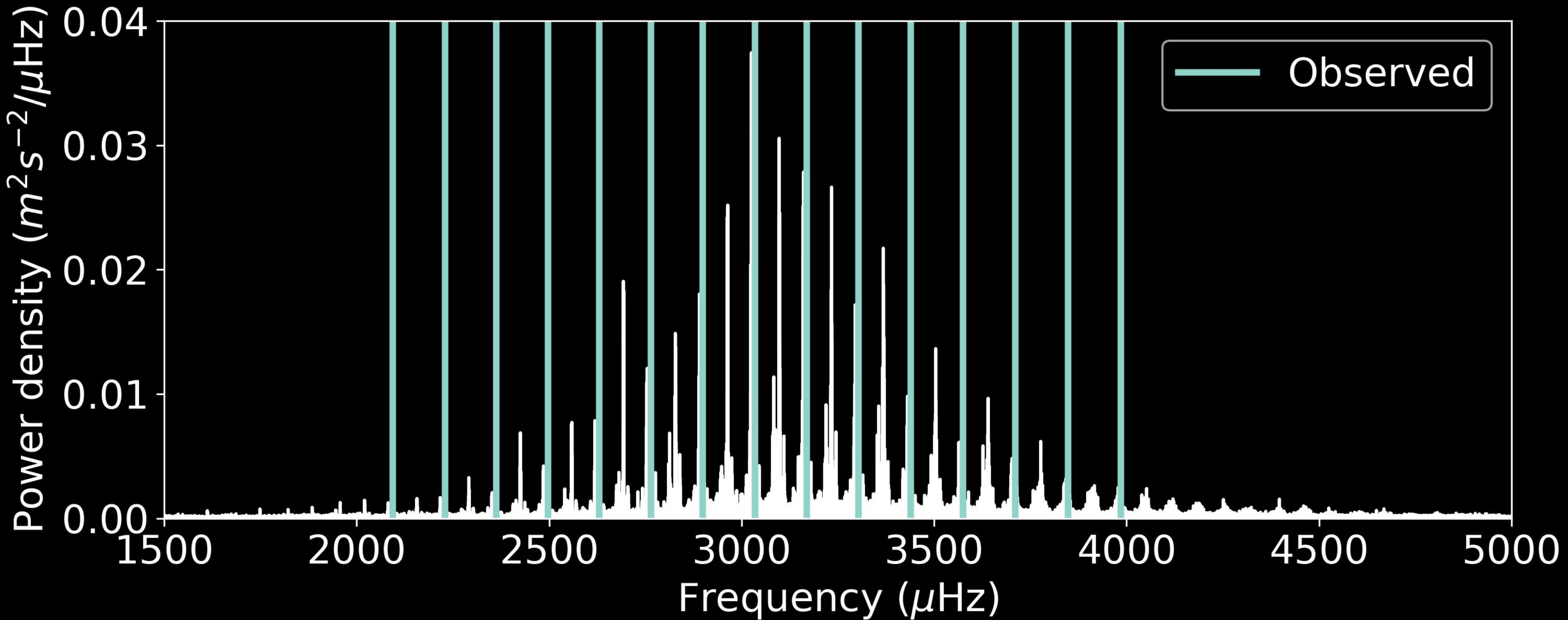
Likelihood

- Classical constraints: full $\chi^2 = \sum_i^N \frac{(D_i - M_i)^2}{\sigma^2}$
 - $\chi_L^2 + \chi_{T_{\text{eff}}}^2 + \chi_{[\text{M/H}]}^2$
- Seismic constraints: reduced $\chi^2 = \frac{1}{N - N_{\text{dof}} - 1} \sum_i^N \frac{(D_i - M_i)^2}{\sigma_i^2}$
 - $\frac{1}{N_{l=0}} \chi_{\nu_{l=0}}^2 + \frac{1}{N_{l=1}} \chi_{\nu_{l=1}}^2 + \frac{1}{N_{l=2}} \chi_{\nu_{l=2}}^2$
 - $\chi_{\nu_{l=0}}^2 + \chi_{\nu_{l=1}}^2 + \chi_{\nu_{l=2}}^2$
 - $\chi_{\nu_{\text{max}}}^2$ (frowned upon, but could be potentially useful)
- Combining classical and seismic constraints with somewhat arbitrary weights (Cunha et al. 2021 for full discussion)

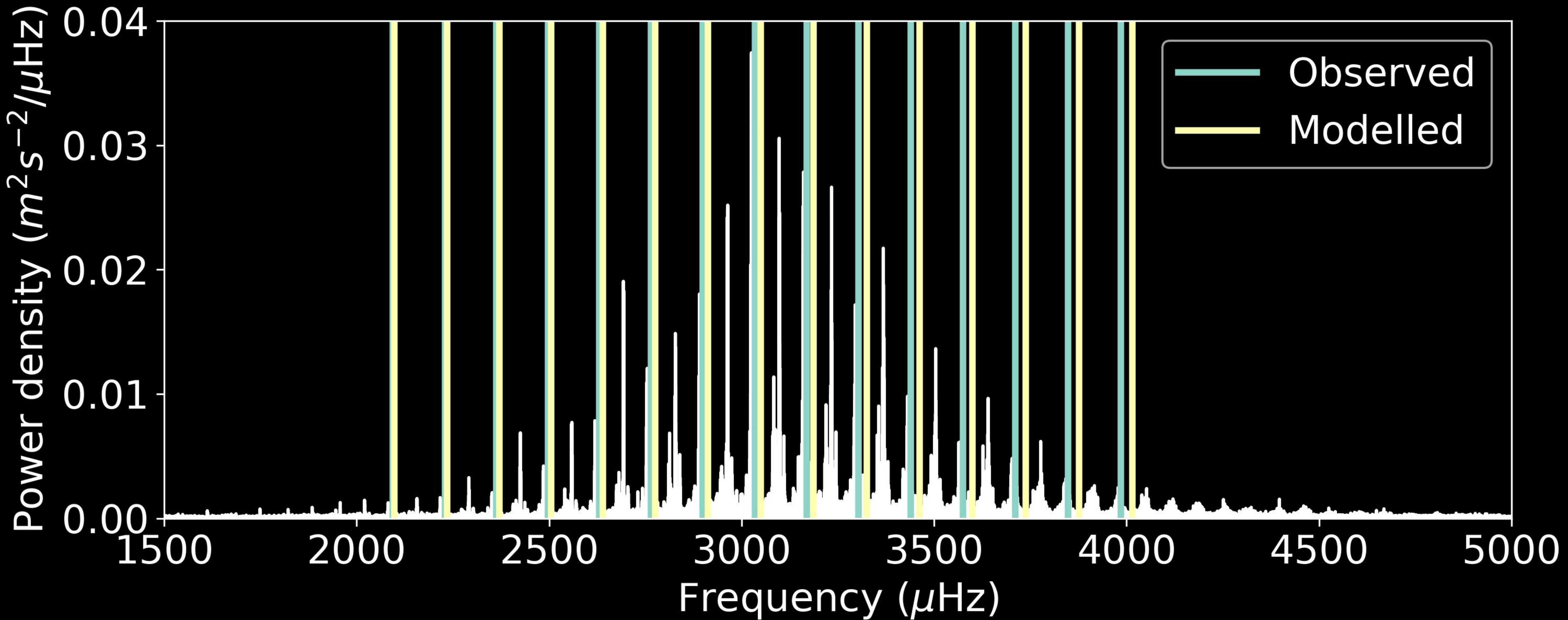
The Surface Effect



The Surface Effect

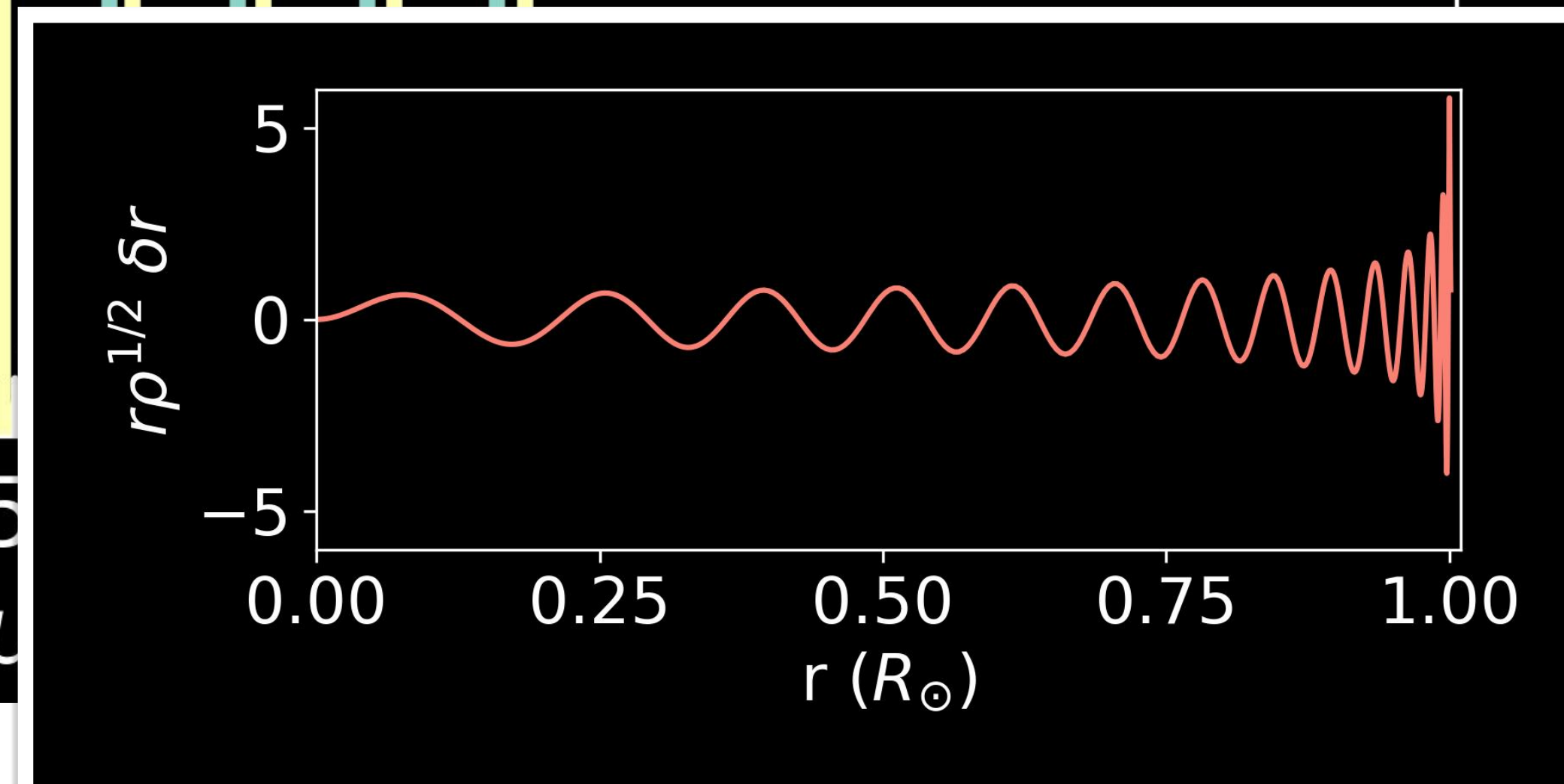
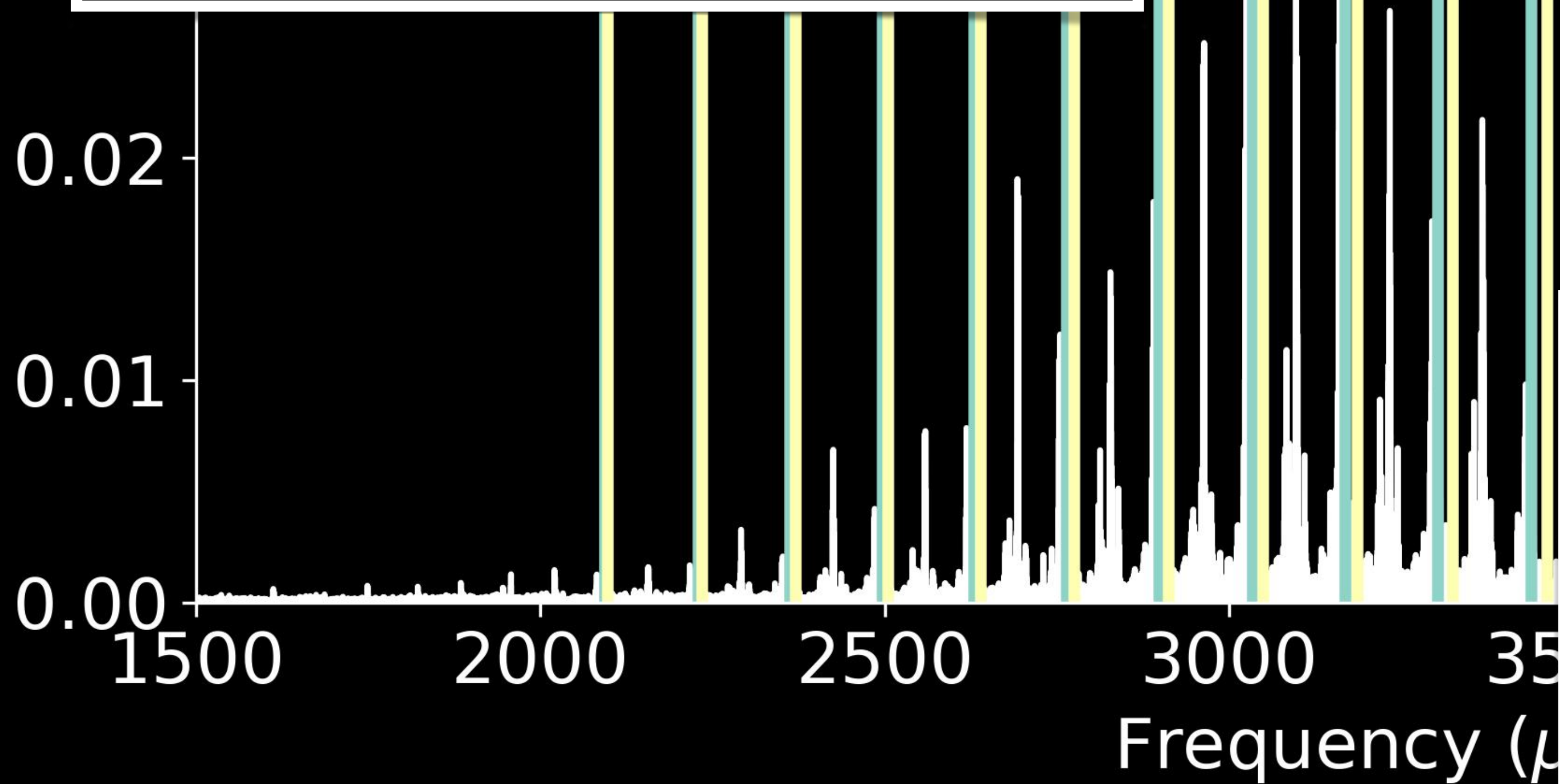
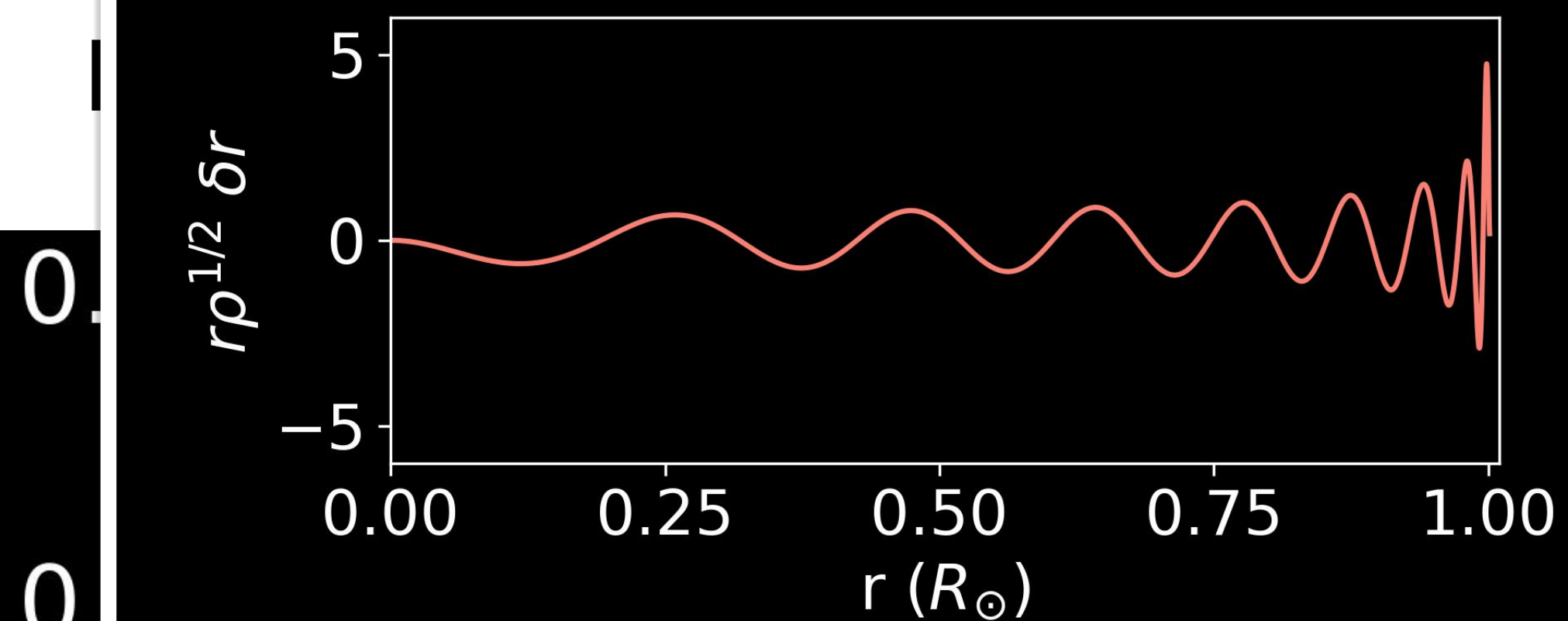


The Surface Effect

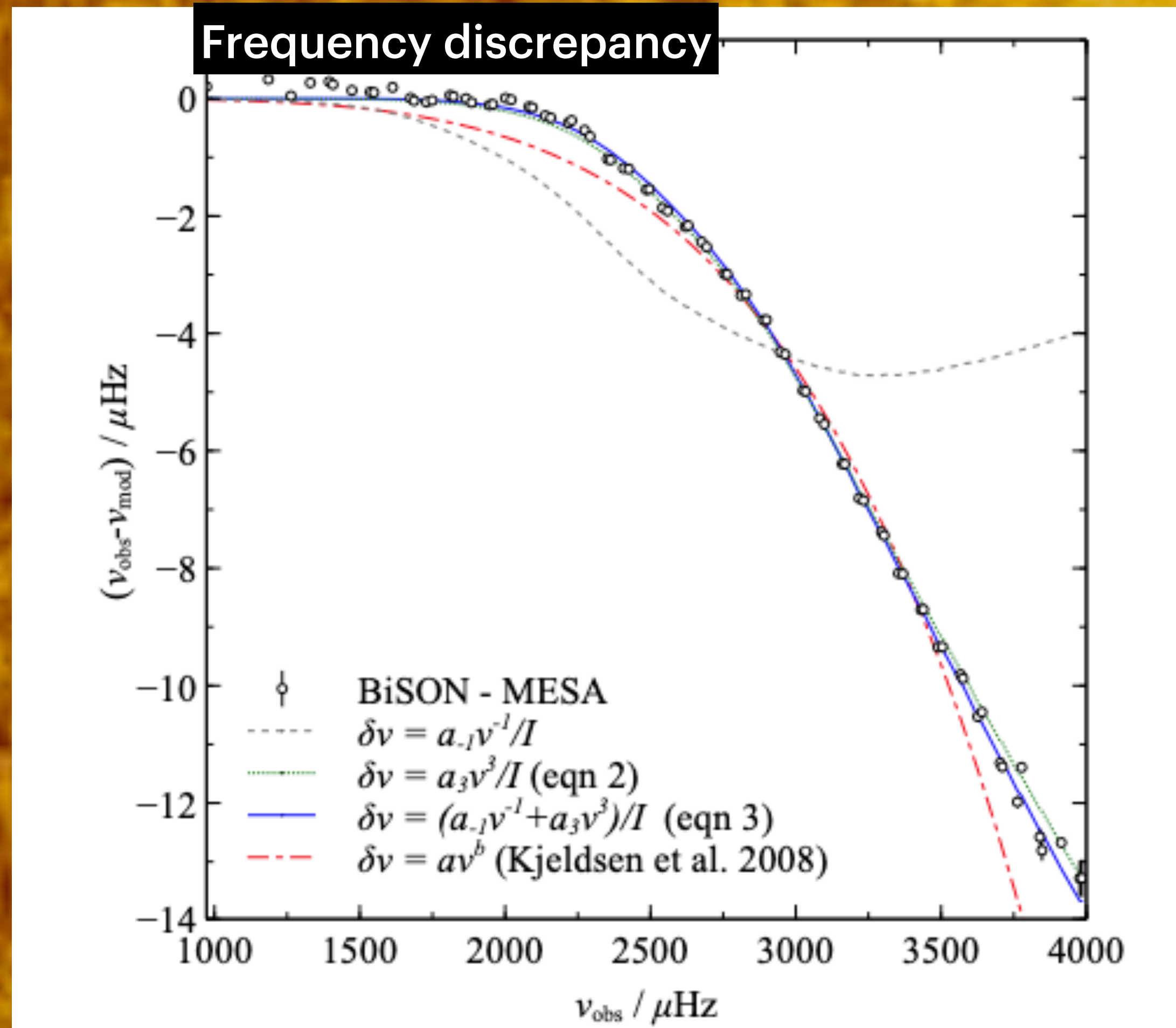


and observed frequencies: the surface effect

Power density ($m^2 s^{-2} / \mu\text{Hz}$)



The Surface Effect



<0.1% * Radius below the surface

$$\delta\nu = \left[a_3(\nu/\nu_{\text{max}})^3 + a_{-1}(\nu/\nu_{\text{max}})^{-1} \right] / I$$

The values of $\hat{a}_3$ and $\hat{a}_{-1}$ are determined as free parameters.

The Surface Effect

- Parameterized model
 - $\delta\nu = \alpha(\nu/\nu_{\max})^\beta$ (Kjeldsen et al. 2008)
 - $\delta\nu = [a_3(\nu/\nu_{\max})^3 + a_{-1}(\nu/\nu_{\max})^{-1}]/I$ (Ball & Gizon 2014)
 - $\delta\nu(\nu_{\max}; a, b, c, d) = a \cdot (g/g_\odot)^b \cdot (T_{\text{eff}}/T_{\text{eff},\odot})^c \cdot (d \cdot [\text{M}/\text{H}] + 1)$ (Li Yaguang et al. 2023)
- ε -matching (Roxburgh et al. 2016)
- Frequency ratios (Roxburgh et al. 2003)
- p-g mixed modes require special treatment
 - Decoupling to p and g modes (Ong et al. 2021)
- Directly modify stellar atmosphere (structural effect)
 - Some modification to the temperature gradient (Li Tanda et al. 2018)
 - Replace with realistic 3D atmospheric simulations (Mosumgaard et al. 2020)
- Account for coupling between pulsation and convection (modal effect)
 - Time dependent convection theory (Houdek et al. 2017)

Posteriors and Parameter Estimation

- $p_i \propto w_i L_i$
 - Parameter estimation: $\hat{\theta} = \sum_i p_i \theta_i$
 - Calculate the cumulative distribution of as a function of parameter of interest (θ : mass, radius, age)
 - $C(\theta) = \sum_{i, x < \theta} p_i(x)$ - determine θ at $C = 0.16, 0.84$ to form 1- σ confidence levels.